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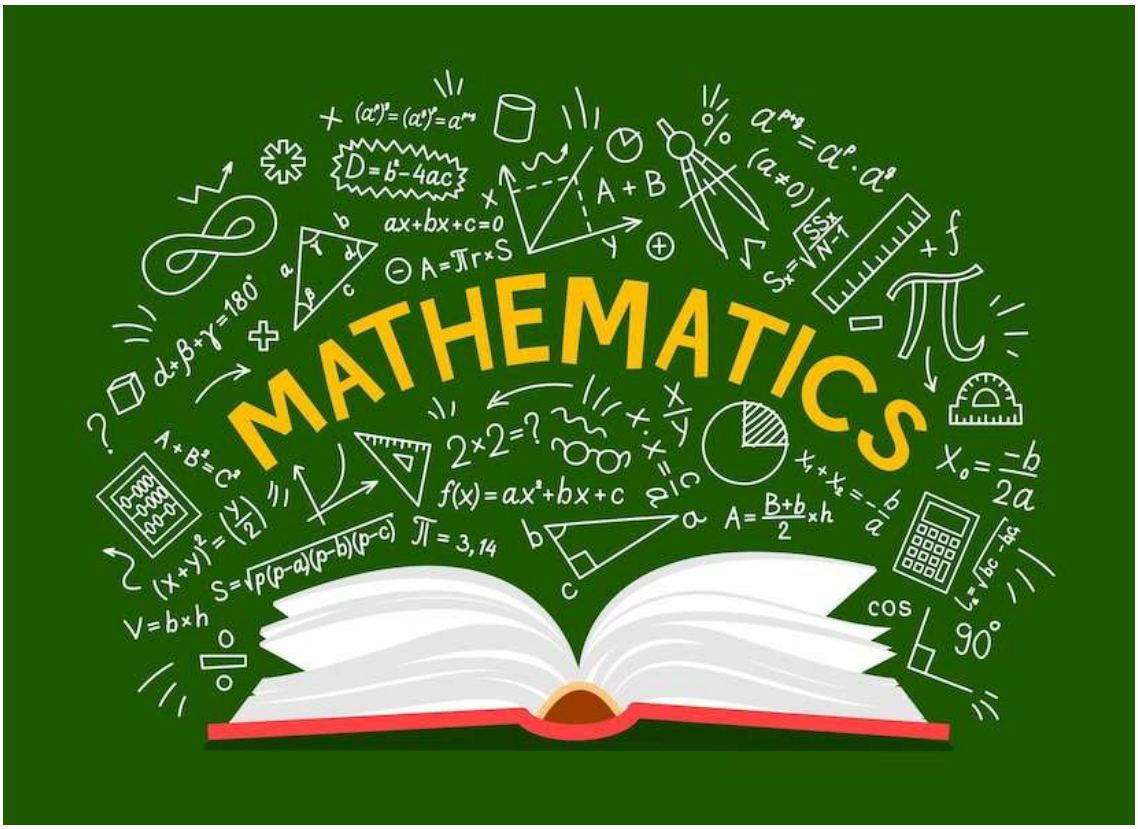

Elective Course

Pedagogy of School Subject

Mathematics

Compiled By

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Mathematics in Relation to Other Academic Discipline

Introduction

Mathematics, as a formal area of teaching and learning, was developed about 5,000 years ago by the Sumerians. They did this at the same time as they developed reading and writing. However, the roots of mathematics go back much more than 5,000 years.

Throughout their history, humans have faced the need to measure and communicate about time, quantity and distance. The early efforts to represent numbers are lost in the past thousand upon thousand years ago. Only one thing is reasonably sure that the symbol for number "1" was single mark either vertical or horizontal like | or and large number could be represented accordingly.

Example: || or = for 2

||| or = for 3 and so on.

The development of reading, writing and formal mathematics 5,000 years ago allowed the codification of math knowledge, formal instruction in mathematics, and began a steady accumulation of mathematical knowledge.

Mathematics as Academic Discipline
A discipline (an organized, formal field of study) such as mathematics tends to be defined by the types of problems it addresses, the methods it uses to address these problems, and the results it has achieved. One way to organize this set of information is to divide it into the following three categories (of course, they overlap each other):

1. **Mathematics as a human endeavour**: For example, consider the math of measurement of time such as years, seasons, months, weeks, days, and so on. Or, consider the measurement of distance, and the different systems of distance measurement that developed throughout the world. Or, think about math in art, dance and music. There is a rich history of human development of mathematics and mathematical uses in our modern society.

2. **Mathematics as a discipline**: You are familiar with lots of academic disciplines such as archaeology, biology, chemistry, economics, history, psychology, sociology, and so on. Mathematics is a broad and deep discipline that is continuing to grow in breadth and depth. Nowadays, a Ph.D. research dissertation in mathematics is typically narrowly focused on definitions, theorems and proofs related to a single problem in a narrow subfield in mathematics.

3. **Mathematics as an interdisciplinary language and tool**: Like reading and writing, math is an important component of learning and "doing" (using one's knowledge) in each academic discipline. Mathematics is such a useful language and tool. So, it is considered one of the "basics" in our formal educational system.
To a large extent, students and many of their teachers tend to define mathematics in terms of what they learn in math courses, and these courses tend to focus on point number 3. The instructional and assessment focus tends to be on basic skills and on solving relatively simple problems using these basic skills. As the three-component discussion given above indicates, this is only part of mathematics.

Relationship between Mathematics and Other Disciplines

Mathematics has not only been utilitarian in its own right, but it has also enriched this world by helping in the development of other disciplines too. It has direct or indirect relationship with the learning of almost all the subjects. This is why it is often called that mathematics is the science of all sciences and art of all arts. Here, we shall discuss the role played by mathematics in the field of physical sciences, biological sciences and social science.

1. Mathematics and Physical Sciences

There is no branch of science in which knowledge of mathematics is not found essential. Different persons have given relationship of mathematics with physical sciences in different fashion. For example, according to Lindsay, "Mathematics is the language of physical sciences and certainly no more marvellous language was ever created by the mind of man". In the word of Mellor J.W., "*It is almost impossible to follow the later developments of physical or general chemistry without a working knowledge of higher mathematics.*" Berthelot has also written in beautiful words that, "Mathematics is the indispensable instrument of all physical researches."

Emphasizing on the closeness of mathematics and physical science, Bacon has written "*Mathematics is the gate and the key of all science. Neglect of mathematics works, injury to all knowledge, since who is ignorant of it, cannot know the other sciences or the things of the world*." Comte presents the relationship of mathematics and physical science as "*All scientific education which does not commence with mathematics is of necessarily defective at its foundation.*"
In this chapter, we shall limit our discussion to the following physical sciences:

1. Physical

2. Chemistry

3. Engineering

1**. Mathematics and Physics**: Mathematics is probably the sole language of physics, and therefore a real understanding of physics impossible without adequate knowledge of mathematics. Every rule and principle of physics is expressed in mathematics language. Take the case of mechanics in physics. Many of the equations are shortened notation where mathematics is used to express relationship between quantities. The equations are precise, concise and easy to remember, e.g., [let initial velocity = u, time = t, final velocity = v, acceleration = a and distance = s, then s = yt, v = u + at and a = (v - u)/t. Similarly, in gravitation, if mass = m, distance from the center of earth = R and gravitational constant = G, then force is given by mathematical formula, i.e., F = G $\frac{m_1 m_2}{R_2}$. Similarly, law of lever which works on the principle of equations tells us that we can carry heavy weight by applying less force through a longer arm, as for example, in the case of handle of a water pump and raising heavy loads with the help of cranes.
In heat, coefficient of linear expansion of different metals, coefficient of cubical expansion, Charles' law of expansion of gases is based upon mathematical calculations. Again, we know that steam can generate powers but the amount of pressure, which the steam would exert and walls will bear, can be only calculated mathematically. In a similar way, graduation of stem of thermometer and then conversion of scales is a mathematical work. Tables of specific heat, latent heat and melting points have been prepared with the knowledge of mathematics. Further if mass = m, specific heat = c and temperature = T, then quantity of heat Q = m.c (T_2-T_1) is given in mathematical form.
In light, we know that light changes its direction in going from a rarer to a denser medium. But to measure the deviation and the relation between the

deviation and the densities of the two media to use this property in making eyeglasses or optical instruments, mathematical knowledge is applied. For example, if focal length = f, distance of object from the mirror or the lens = u and distance of the image from the mirror or lens = v, then $1/f = 1/u + 1/v$ for spherical mirror and $1/f = 1/v - 1/u$ for spherical lens.
Electronic is new field in physics, which is wholly based on mathematics. Microwave transmission, ultrasonic hammers and drills, solid-state semiconductors, multichannel colour TV, video and computer are but a few applications of physics made possible by mathematical use. Solution of numerical problems in physics reveals the value of mathematics in learning physics.
In short, algebraic equations, graphs, units, geometry, coordinate geometry, calculus, logarithms, differential equations, exponents and statistics are some of the useful tools of physics.

2. Mathematics and Chemistry: Mathematics and chemistry are very much interdependent.
For example, chemical combinations are governed by certain mathematical laws, all chemical compounds are found after huge mathematical calculation. Take the case of water, two atoms of hydrogen are needed for each atom of oxygen. In the physical and chemical properties of matter, mathematics knowledge is utilized.

In the estimation of element in organic compounds, knowledge of percentage and ratio is utilized. Molecular weights of organic compounds are calculated mathematically. Similarly, in classification of elements, in chemical bonding and in chemical reaction, knowledge of mathematics is essential.

Mathematical law is applied in the structure of atom. Valences of elements have a mathematical base. Chemical equations are governed by mathematical principles. How much heat is generated or required in various chemical actions is another mathematical problem.

In the preparation of different gases and chemical products like polish, cream, salt, acids, medicines, bleaching, powder, etc., we need exact measurement in terms of weight, ratio, etc. Similarly, in the preparation of cooking gas, burning fuels and in the fuels of rockets and bomb, mathematics plays its valuable contribution.

3. Mathematics and Engineering: Engineering is probably incomplete without mathematics. Mathematical calculations have given a sound footing to the engineering laws and principles. For the understanding of various activities of engineering, e.g., surveying leveling, designing, aeronautical, civil construction, electrical, mechanical, industrial or metallurgical, the knowledge of mathematics is essential.

For the construction of bridge over a stream, the engineer will have to consider the width of the stream. With the help of knowledge of trigonometry, he has to take into consideration the number of pillars and durability of the material to be used with the help of ratio and proportion knowledge of mathematics. Similarly, in the construction of buildings, he takes the help of geometry for designing purpose. In making aeroplanes, railways, buses and various other mechanical, industrial or electrical goods, calculator and computers, it is necessary to have some mathematical background.

II. Mathematics and Biological Sciences

Mathematics and biological sciences are related to each other to a greater extent. Advances have been made in physiology, genetics, heredity, nutrition, growth, maturation, metabolism, fatigue and many other biological and physiological studies with the help of mathematics.

The caloric and nutritive values of the food articles are calculated with the help of mathematics. Rate of respiration, transpiration and supply of water in living bodies are interpreted mathematically.

Statistical methods are applied to the results of biological experiments. For example, the growth in weight of infants up to nine months is given by the formula, $\text{Log}\frac{x}{341.5-x} = k\,(t1.66)$, where x is the weight in ounces and t is the number of months. Similarly, Schultz-Borisoff law with regard to the action of enzymes such as pepsin and rennin is expressed by the formula $x = K\sqrt{F}.gt$, where x is amount of substance transferred, t is the time of transformation, f is the concentration of enzyme, g is the initial concentration of substance (e.g., albumen of milk) and K is the constant.

Biophysics, biotechnology and biochemistry have attained the rank of independent science and Biomathematics has come to be a recognized field of study for biologists.

The expression and the calculation of results of the Mendelian Laws of Heredity are another example of the use of language of the ordinary high school mathematics.

In medical science, the diagnosis and remedial treatment of patient is based on mathematical knowledge. For example, measuring of temperature, blood pressure, urine and stool tests or knowing about the deficiency or excess of any mineral or substances in the body, functioning of X-ray machines or other medical instrument depend on mathematics knowledge. Again, allopathic, homeopathic or Ayurvedic doctors utilize the knowledge of mathematics in preparation and prescription of doses of medicines. Again, knowledge of graph helps the doctors to understand rise or fall of temperature, blood pressure, etc. in a better way than any other method

III. Mathematics and Social Sciences

Social sciences are much related and dependent on mathematics. Below we shall discuss the relationship of mathematics with the following social sciences:

1. Economics

2. Psychology

3. History

4. Geography

5. Philosophy

6. Statistics

7. Music and Dance

8. Fine

9. Logic

10. Civics

1. Mathematics and Economics: Mathematical methods are employed in the laws of diminishing utility and marginal utility. Economics deals with the actual amount of money, production, capital, labour utility, satisfaction, rent, profit, etc. whereas knowledge of mathematics is applied in trade cycle, trend of export and imports, volume of trade, public health, tariff, expenditure of public finance, population trends, etc. Statistical method, graphs bar diagrams, etc. are used.

In life insurance, fire insurance and insurance from all kinds, the theory of probability is applied. The production, purchase, sale and distribution of various things, sale-purchase of property and exchange of different currencies belonging to different nations, etc. are decided mathematically.

Labour and capital relationship depends on the knowledge of mathematics. In judging the justice of progressive income tax, collection of all taxes, and commercial system like banks depend totally on mathematics knowledge.

Knowledge of mathematics is very useful for businessman, because he faces problems involving simple and compound interest, instalment payment, share and discount and of maintaining accounts.

In magazine and newspapers, information given in the form of graphs are more effective in cases such as agriculture, minerals, industrial products, means of transport, imports, population, educational progress, rate of growth of income and expenditure. Various magazines, newspapers and bulletins present figures on accidents, government budgets, markets, crops, employment, rates of increase and decrease in index numbers, and production data which are easily understood with the help of mathematical knowledge.

Similarly, in country's economic planning including five-year plans, new technology know-how and management, monetary and fiscal policies, and prices mechanism, mathematics plays a great role. Again, one cannot get much profit out of the up-to-date journals on Economics if one does not know the modern mathematical and statistical methods of dealing with the subjects of mathematics.

2. Mathematics and Psychology: Mathematics helps in the clear understanding of the subject of psychology. That is why Herbert Spencer has said, "it is not only possible but necessary that mathematics should be applied to psychology." There are certain concepts in psychology such as intelligence quotient, mean, median, mode, standard deviation, correlation, etc. in which knowledge of mathematics and statistics is applied.

Again, statistical analysis is the only scientific method of attacking social and psychological phenomena. For example, human individual and their relations to

one another cannot be investigated in the same manner as white rats or guinea pigs. Individual differences among human beings with regard to their behaviour under certain conditions are so great that no scientific conclusion is possible if the experiment is confined to single individual or a small selected group. Therefore, when such experiments are made, they have to be made on large groups taken at random. It is mathematical problem solved with the help of coefficient of correlation. For the interpretation of such results, statistical analysis is necessary. Not only this, but in the solution of many other psychological, economic and social problems, coefficient is widely used.

3. Mathematics and History: History is the study of past events. Extent organization and duration of the different empires of the past can only be understood and appreciated with the knowledge of the calendar in mathematics, will not information given in the form of graphs be more effective in such cases as growth and downfall of empires and prevalence of wars and famines?

4. Mathematics and Geography: Geography is the science, which described earth in the universe. There are certain topics in geography such as position of the earth, formation of days and nights, change of seasons, lunar and solar eclipse, tides, current movement of earth and winds, falling of rains, factors influencing the climate of a region, boundary of different states, region, countries, etc. which can only be understood with the sound knowledge of mathematics. Similarly, knowledge of geometry helps the students to draw and understand geographical in geography.

5. Mathematics and Philosophy: On a superficial examination of the subject-matter dealt within mathematics and philosophy, one might wonder about what is the possible relation there could be between such widely different subject as mathematics deals with things which is simple, clear and definite whereas philosophy is defined as science and investigates the ultimate reality of things. Moreover, the words 'ultimate' and 'reality' are not simple, clear and definite as these are matter of depute among philosophers.
But even then the relationship between the two subjects is possible to the greater extent. The function of the mathematics in the development of philosophical thought has been very neatly put by the great educationist Herbert. He said, "The real finisher of our education is Philosophy, but it is the office of mathematics to ward off the dangers of philosophy." By warding off the dangers, mathematics puts the philosophers on the right path of acquiring true knowledge. The history of philosophy makes it clear. It will show that at every stage, every advance in the development of the mathematics was followed

almost as a directed consequence by a corresponding advance in philosophical thought. Philosophy owes a great deal to men like Pascal, Descartes, Russell and Leibnitz who were all great mathematicians first.

6. Mathematics and Statistics: Mathematics plays a great role in understanding statistics Various graphs, bar diagrams, frequency polygons, histogram and gives enable us to understand the relation between variable quantities. They are used in the solution of social, governmental, economic and scientific problems. Many magazines, newspapers, bulletins and research theses are full of statistical analysis such as mean, median, standard deviation, correlation. factor analysis, index number, etc. which are used in the solution of many social and psychological phenomena.

7. Mathematics and Music and Dance: Music and dance are much related to mathematics Almost all the musician instruments, e.g., harmonium, guitar, sitar, violin, etc. are played according to some set principles of mathematics. Again, for ascending or descending order of tone and pitch of music, mathematical knowledge is required. In dance too, for the movements of limbs, postures, taking of steps and responding to the tunes, knowledge of mathematics is essential.

No one is better qualified to speak on music and mathematics than the great scientist and musician Helmholtz who has this to say, "Mathematics and music are the most sharply contrasted fields of scientific activity which can be found, and yet related, and supporting each other."

8. Mathematics and Fine Art: Rhythms, proportion, balance and symmetry, which are essentially mathematical in character, are basis to any field of fine arts. With the application of Mathematics, we can make the teaching and learning of fine arts interesting and real. Drawing is of immense importance in the study of all the branches of the fine arts. The preparation of charts and pictures require 'mathematical knowledge

It is only an engineer, who realizes the difficulty of intermeeting laws of similarity, symmetry and proportionality into brick, mortar and wood, that can the begun appreciate fully the beauty of a temple tower. The musician who knows the scientific and mathematical basis of music can appreciate and appraise a piece of music much better than another who does not know the mathematics.

Greeks, the greatest geometrier of the age, were also the most successful in art sculpture. They knew how to weave their knowledge of geometrical proportion and symmetry into their great work of art. Again, Pythagoras, the great

mathematician observed that "musical strings of equal lengths stretched by weights having the proportion of 1/2, 2/3, 3/4, produced intervals which were an octave, a fifth and a fourth. Harmony, therefore, depends on musical proportion; it is nothing but a mysterious numerical relation. Where harmony is, there are numbers". In the words of Boltzmann, "the simplicity, the indispensableness of each word, each little dash, that among all artists raises the mathematician nearest to the world creator. It establishes a sublimity which is found in other art and music."

9. Mathematics and Logic: Logic is the scientific study of the conditions of accurate thinking and valid interference. According to D'Alembert, "Geometry is a practical logic, because in it, rules of reasoning are applied in the most simple and sensible manner." Mathematics is helpful in training or teaching logic on right lines because symbols and methods used in mathematics can be transferred to the study of logical laws. For example, in mathematics $x < y$ means x is less than y. In logic, the meaning has been extended. Let x denote the class denoted by buffaloes and y denoted by the animals, then x, y is easily interpreted to result, i.c., all buffaloes are animals, then x, y is easily interpreted to mean that x is included in y. In this way, we see that the meaning of mathematical symbols have extended to represent the relationship of proposition in logic. Hence, the justifications for the following statements. "Mathematics is but the higher development of Symbolic Logic" as per Whetham. "The two great components of the critical movement, though distinct in origin and following separate paths, are found to converge at last in the thesis: Symbolic Logic is Mathematics. Mathematics is Symbolic Logic. The two are one" as said by Keyser.

10. Mathematics and Civics: In civics also, knowledge of mathematics is utilized. In village. town or city, how many houses are there, what are the sources of income of village panchayat, municipality, what are the expenditure of these bodies, etc., these can only be known if one has sound knowledge of mathematics. In democratic country, at the time of election of the level bodies, e.g., Panchayat, municipality, etc. we have to count the voters. Similarly, at the time of election to the state assemblies and Lok Sabha, there is need of census to know the number of voters, number of candidates, number of constituencies. defeat and victory of candidate. At the time of election of President of India, we have to calculate the value of votes and member of legislative assembly of different states and members of parliament which depends on mathematical knowledge

For economic planning, budgeting and other saving schemes for the betterment of citizens, we need the knowledge of mathematics.

IV. Relation of Mathematics with Some Other Subjects

There are some other subjects besides the above which are also closely related with the knowledge of mathematics. There are discussed below.

1. Mathematics and Agriculture: In agriculture, while measuring land or area, average income, production per unit area, cost of labour, time and work, seed rate, fertilizers rate or electric bill for running tube well, for purchasing or selling of land, purchasing and selling of agriculture implements, vegetable, crop, or maintaining the account of money borrowed from bank, one uses the knowledge of mathematics.

Similarly, with the knowledge of graph in mathematics, we can very well know about the yield per acre for different years for a particular crop of Millet in India or for different crop. Knowledge of mathematics is also helpful in the field of agriculture while planning and growing different crops according to their season and giving water to land from canal.

2. Mathematics and Language: Mathematical statements cannot be framed without sound knowledge of language Mathematics, which is the language of number, sign, symbols and formula, is dependent on languages for its exactness and accuracy in communication. Historical events and biographies of mathematician provide an excellent material for reading. Similarly, in grammar portion of the language, we apply some rules such as application of comma, fullstop, question mark and all these are governed by mathematical knowledge. Again, in poetry, we have to be very conscious about its symmetry and regularity to raise its aesthetic value.

3. Mathematics and Physical Education: In physical education, we can understand the structure of human body, temperature of human body, blood pressure, heartbeat, pulse rate and balance diet for different persons of different age with the help of mathematics. In games, sports and athletes, one has to measure, count and weight according to some standard, set for evaluation purpose.

4. Mathematics and Work Experience (or Craft): There is no craft in which we utilize the knowledge of mathematics. For example, in embroidery, clay

modelling, wood work, cane craft or leather craft, we need mathematics for drawing, designing purpose and for weighing, counting or making calculation regarding time, work and mode of payment of good.

Take the handloom craft, problems like how much cotton or thread is required for making particular design, what is the number of threads used, what type of geometrical design is required, etc. can only be known with the help of mathematics. For making ink, soap or polish, we have to prepare a mixture according to some set ratio. In gardening also, for floriculture or distribution of land for ploughing, manuring and watering.

Practice Questions

1. Write notes on the following:

(a) Relation of Mathematics with Physics.

(b) Relation of Mathematics with Agriculture.

(c) Relationship of Mathematics with Fine Arts.

(d) Relationship of Mathematics with Chemistry.

2. How can you relate mathematics with other Academic disciplines such as Physics and Geography?

Place of Mathematic in the Present School Curriculum

Introduction

In education, the importance and the place of a particular subject depends on the fact that "to what extent the subject is helpful in achieving the aims of the education" If any subject is more useful for achieving educational objectives, then its importance increases accordingly. Since ancient times, mathematics has played a vital role in "achieving aims of education, as compared to others. Present age is the age of science and information. Whatever, technological and physical progress being made, shall be correspondent to the role of mathematics. Being so important. -What place should be given to mathematics in the curriculum?"

Place of Mathematics

Mathematics is a fundamental part of human thought and logic, and integral to attempts at understanding the world and ourselves. Mathematics provides an effective way of building mental discipline, and encourages logical reasoning and mental rigor. In addition, mathematical knowledge plays a crucial role in understanding the contents of other school subjects such as science, social studies, and even music and art.

Mathematics, as a subject, has its place in Indian school system since pre-independence era and British system of schooling started in three stages primary, middle and high schools - and took up measures for the promotion of the indigenous system of education which included: **At lower level** (i) Arithmetic written and mental arithmetic (ii) Bazar and Zamindari accounts and simple mensuration; **At middle schools level:** Arithmetic, Theory of Surveying, Bazar and Zamindari accounts, Handling of money matters and Geometry and Mensuration; M.E. Madrasa and Sanskrit Middle School had a common curriculum of study for Arithmetic/Mathematics.

Later, Basic Education Act (1954) also emphasized the curriculum for primary level with Arithmetic, Mental Arithmetic, Accounts, Jama Kharach (savings and expenditure) and reading of clock.

National Policy of Education (1968), with recommendations of Kothari Commission, made Mathematics and Science compulsory core subject at Middle and Secondary stage. Mathematics and Science were greatly stressed. Accordingly, General Mathematics was compulsory subject up to Class X, and at Secondary level, Advance Mathematics was there as an optional subject.

After NPE 1955 and POA 1992 major reformation in school education was attempted with respect to Science and Mathematics. The curriculum was made heavy and modernized at middle secondary level, and to handle the subject, separate Science Gradate teachers were appointed at the middle and secondary stage. Science and Mathematics kits were supplied to schools under Operation Black Board Scheme to learn the subject by doing

NCF (2005) reiterated the significance of mathematics in Indian school system, and understood the need of making the subject more interesting and burden-free. The s mathematics education according to NCF (2005) was that:

- Children learn to enjoy Mathematics rather than fear it

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- Children learn important Mathematics: Mathematics is more than formulas and mechanical procedures.
- Children see Mathematics as something to talk about to communicate through, to discuss among them and to work together on
- Children pose and solve meaningful problems
- Children use abstractions to perceive relationships, to see structures, to reason out things, to argue the truth or falsity of statements
- Children understand the basic structure of Mathematics: Arithmetic, Algebra, Geometry and Trigonometry, the basic content areas of school Mathematics; all offer a methodology for abstraction, structuration and generalization
- Teachers engage every child in class with the conviction that everyone can learn Mathematics.

In Mathematics, the new syllabi emphasize reasoning and conceptual grasp at every stage. In primary Mathematics, weightage has been projected to areas like shapes, spatial understanding, pattern, measurement and data handling. And Mathematics modelling, data analysis and interpretation provided at secondary stage set the frame to perceive Mathematics as discipline. At higher secondary stage, constructivism and problem solving form the twin objectives of syllabus Emphasis on activity rather than rote memorization of facts and formulae continue through all stages.

But there is another part of intellectuals who believe that there is no need to make mathematics as a compulsory subject till secondary stage. According to them, it should be made optional after middle stage of school. Till then, students get the knowledge of all the basic. Functions of mathematics in daily life. They put forth some arguments against making mathematics a compulsory subject up to high school stage:

1. **Learning of mathematics requires special mental abilities:** It is believed that the study of mathematics requires special ability, and this being inborn and innate, therefore only more promising student should be encouraged to study mathematics. As it is not everybody's cup of tea, therefore everybody should not be compelled to study this subject.

2. **Mathematics is tough subject:** It is believed that mathematics is a tough subject and generally result in mathematics is poor. Therefore, they are of the view that mathematics may be made an optional subject.

3. **Other subjects can cover up disciplinary value of mathematics:** According to their views, if mathematics is taught because of its disciplinary value, i.e., because it helps in the development of reasoning and thinking power, even then it is not true as students studying subjects other than mathematics are also having good mental faculties. Hence, mathematics need not be taught compulsorily.

4. **Not significant transfer of learning:** It is believed that mathematics helps in the study of other subjects, but if actually seen mathematics has some relation with

science, arts and drawing only. Hence, no transfer of learning is possible in the learning of other subjects.

5. **Does not help all students in preparation for higher studies:** It is said that the study of mathematics helps in the preparation for the higher studies. But according to critics, mathematics is useful only to those students who wish to study higher mathematics in engineering or some science subject at the college or university stage. Mathematics is not going to help that majority of students who want to study arts subjects at college or university level. Since only a small percentage of the students go up to the university stage and take up mathematics, therefore there is no need of making mathematics compulsory up to high school stage.

6. **Relatively small number students go for vocational courses:** No doubt, study of mathematics is useful in some vocations like engineering, banking, accountancy, statistics, surveying, teaching, etc., but very few students join such professions. Thus, the requirements of such few professions need not dominate the school curriculum and there is no need of burdening the brains of other with mathematical facts, which are divorced from life and contrary to nature.

7. **Other countries have mathematics as optional subject:** When in many countries of the world, mathematics is taught compulsory only up to middle stage, then why to make it compulsory up to high school stage in India.

8. **Sufficient till middle stage of school:** Mathematics is taught due to its practical value. But critics say that knowledge of mathematics which the child gets up to middle stage is also sufficient for our everyday life. Moreover, what we teach in mathematics is divorced from actual life. Therefore, mathematics teaching should not be made compulsory up to high school stage.

9. **Dull and dry subject:** Mathematics is generally regarded as dull, dry, and boring subject. Very few students have natural aptitude and genuine love for the subject. Therefore, it will be unpsychological to make this subject compulsory for those students who are not really interested in its study.

But there is other side of the picture. Those who are in favour of making mathematics compulsory up to high school stage say that viewpoints of the critics are unreasonable. So following are the arguments in favour of Mathematics as compulsory subjects:

1. **More of general intelligence (G) rather than specific intelligence (S):** As far as special ability for mathematics is concerned, it has been proved with help of experiment that mathematics, more of general intelligence (G) is needed than of specific intelligence (S) Therefore, there is no truth in saying that only intelligent students should be encouraged to learn mathematics.

2. **Teachers are responsible for making mathematics tough:** It is also wrong that mathematics is a tough subject. The teachers often lay stress on memory work. So, the teachers are responsible for creating a distaste and hatred for the subject, and degenerate understanding. It is the method of teaching rather than the subject itself wherein lies the defect. The formulas the rules and principles need not be memorized

If arrived properly, they can be easily rediscovered when forgotten. The teacher can make the subject simple and easy by improving the quality of reasoning and thinking power of the students.

3. **Mathematics develops the power of thinking and reasoning:** It is said beyond doubt that mathematics develops the power of thinking and reasoning. It gives mental exercise best fitted for strengthening the faculties of the brain. The logic employed is simple, exact, accurate, true and useful. According to mathematician Young. "It is the only subject that encourages and develops logical thinking. It enables the students to discriminate between essentials and non-essentials. It helps them to sift facts, to draw conclusions tersely and without ambiguity and that it is a subject by which they may learn what is meant by rigid reasoning..."

4. **Prepares for higher education:** It is not waste of time to cater to the needs of the highe education at the school stage because education at one stage must aim to prepare the child for the education at the higher stage. It is true that the child may not need the study of certain topics in mathematics at the present stage but it is equally true that he would facet the ignorance of these topics as a major handicap in gaining knowledge in the higher classes. Therefore, it is correct to say that study of mathematics helps in the preparation for the higher studies.

5. **Provides broader view about life:** No doubt, everybody who is studying mathematics the school is not going to become engineer, accountant, mathematics teacher and statistician, but at such an early stage of education, it is difficult to know who is going to be an engineer or a banker or statistician. Therefore, it is wrong to say that by making mathematics compulsory up to high school stage, we are going to burden the brains of students. Instead of this, we are providing them a broad view of what they are capable of achieving in their future life.

6. **Level of other country's curriculum is broader than Indian curriculum:** In those countries where mathematics is compulsory only up to middle stage, the standard of mathematics teaching is quite higher as compared to our country. What their students study up to middle stage is more than what our students learns up to high school stage Also, in those countries, the mental and chronological age of the child studying in eighth class is just equivalent to the student studying in tenth class in our country. In other words at high school stage, in foreign countries, the pupils are two years older. In those countries, there are adequate facilities and opportunities for guidance and counselling. and the parents are also well educated and can guide their children. But in India, at high school stage, children are too young to choose vocation for themselves, and many parents are also illiterate and unable to guide their children. Parents, due to their low social-economic status, are not definite, whether they will be able to send their children for higher studies or for some good profession. Therefore, in India, it is the duty of the school to give to the high school students a broad view of what may lie in their future and to give them a course whereby they may be better able to judge for themselves as to their best line of action and not to make them regret about their early choice.

7. **Mathematics is not a dull subject:** There is also misunderstanding in the minds of the people that mathematics is dry, dull, dreadful and boring subject. As far as making the subject tasty and interesting, much depends upon the teachers and their methods of teaching than the subject itself. As few teachers give more emphasis on memorization and neglect the cultivation of reasoning and thinking power of the student, the students do not love this subject. Therefore, teachers are blamed for this affair and not the subject of mathematics.

So, it is quite clear from the above discourse that a subject, which is so valuable and psychologically based and so closely connected with our daily life, is justified to be included in the school curriculum and its teaching should be made compulsory up to high school stage. High and senior secondary education will remain incomplete and will not be comprehensive if mathematics is excluded from high school stage. On the importance and place of mathematics in the school curriculum, even the noted Indian Education Commission (1964-66) has recommended "Science and Mathematics should be taught on a compulsory basis to all pupils as a part of general education during the first ten years of schooling. In addition, there should be provision of special courses in these subjects at the secondary stage, for students of more than average ability.“

National Policy on Education (1986) of Government of India has also emphasized in strong words for making mathematics compulsory up to ten years schooling, and making it optional and specialized subject to the other subject of study at plus one stage of school education.

We can conclude our argument by saying that every school subject is too little equipped to handle the concept and terms of modern science and computer technology. Mathematics is, therefore, the only available tool for twenty-first century learning.

Practice Questions

1. Should mathematics be a compulsory subject in our High classes? Debate in favour and against the motion.
2. Elaborate the place of mathematics in school curriculum while referring to government policies and Commission

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Meaning, Nature and Scope of Mathematics

Introduction

Mathematics has been applauded as one of the alpha-prima of human intellectual ventures throughout the centuries, From the very beginning, mathematics has been a living and growing Intellectual pursuit. It has its roots in every day activities and forms, unit basic structure of our highly advanced technological developments It also offer opportunities for inception and direction of new lines of creative ideas and thoughts It exhibits connections between things which can be visualized only through the agency of human reason. Now, let us try to understand something about what is mathematics and what does it encompass.

Meaning of Mathematics

Mathematics is a way of organizing our experiences of the world around us. It enables us to communicate and make sense of our experiences. We can solve a range of practical tasks and real-life problems. Mathematics is quantitative aspect of human life as it deals with the factors (facts, knowledge, relationships and experiences) are empirical in nature. It deals with relationship of magnitudes and directions of quantitative and qualitative attributes.

The word "Mathematics" has been used in two distinct and different senses, viz,

- Practical sense, i.e., method used to solve the problems of quantity, space, order, etc. and
- Abstract sense, i.e., a set of laws or generalizations of truths that are discovered. Different dictionaries and mathematicians have defined mathematics differently, stressing its various characteristics. Here are few representative definitions.

According to ***Bhargava's Standard Illustrated Dictionary*** (1990), "Mathematics is the science space and number.“

Chamber's Twentieth Century Dictionary (1987) says "Mathematics is the science of magnitude and number and of all their relations. “

Webster's New World Dictionary (1973) defines mathematics as "the science dealing with quantities, forms, etc. and their relationships by the use of numbers and symbols.“

Goods Dictionary of Education (1984) has defined "Mathematics as the science of measurement, quantity and magnitude.“

In the words of ***P.A.M. Dirac***, "Mathematics is the tool specially suited for dealing with abstract concepts of any kind and there is no limit to its power in this field." In other words, as in mathematics, we find results at the abstract level with the help of process of reasoning. Therefore, mathematics may be regarded as a science of abstract form.

According to Lindsay, "Mathematics is the language of physical science and certainty. No more marvellous language was ever created by the man.“

As expressed by ***Hindu Mathematicians***, "mathematics (Ganita) means the science of calculation.“

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From the above opinions, we conclude that Mathematics is a science of quantity (amount, size, portion) and space (time, interval, distance). It deals with the questions and problems involving, size, portion, area, time, interval, distance, etc. It is also a science of calculation involving the use of number and symbols. It deals with the relationship between magnitudes. It deals with the numerical part of man's life. In this way' it is a systematic, organized and exact branch of science, which deals with abstract concepts.

On the basis of above speculation, the following points can be concluded:

- Mathematics is study of quantitative facts and relationships.
- Mathematics is abstract form of science.
- Mathematics is science of logical reasoning.
- Mathematics is inductive science.
- Mathematics is science of space and numbers.

Nature of Mathematics

Mathematics, like everything else that man has created, exists fulfill certain human needs and desires. It is very difficult to say at what point of time in the history of mankind, and in which part of the world, mathematics had its birth. The fact that it has been steadily pursued for so many centuries, that it has attracted ever-increasing attention and that it is now the dominant intellectual interest of mankind shows that it appeals very powerfully, to mankind. This conclusion is borne out by everything that we know about the origin of mathematics. More than 2,000 years before the beginning of the Christian era, both the Babylonians and the Egyptians were in possession of systematic methods of measuring space and time. They had the knowledge of rudimentary geometry and rudimentary astronomy. This rudimentary mathematics was formulated to meet the practical needs of an agricultural population. Their geometry resulted from the measurements made necessary by problems of land surveying. Units of measurement, originally a stone or a vessel of water for weight eventually became uniform over considerable areas under names, which are now almost forgotten.

Zero was defined and this at once led to positional notations for whole numbers and later to the same notation for fractions. The place value system, which eventually developed, was a gift of this period. These achievements and many more of a similar nature are the triumph of the human spirit.

Mathematics is something that the man has himself created to meet the cultural demands of time. Nearly every primitive tribe invented words to represent numbers. But it was only when ancient civilizations such as the Sumerians, Babylonian, the Chinese and the Mayan developed trade, architecture, taxation and other civilized contracts that the number systems were developed.

Thus, mathematics has grown into one of the most important cultural components of our society: the modern way of life would hardly have been possible without mathematics. Imagine trying to get through the day without using a number in some manner or the other. If a person lacks the ability to compute, he is as good as crippled. For instance, we need to know the time and tell the same. Telling the time is difficult and yet nearly everyone learns it. A degree of estimation, not only in money but also in weights and measures, is very

important. Many of our daily routine chores involve sorting, ordering and organizing processes. For travel, reading of maps and diagrams, interpreting scales becomes an essential part of our intellectual equipment.

Knowledge of mathematics is useful to understand and interpret matters such as income tax and the information presented to us by the mass media in numerical form or in the form of graph helps us to understand the use of phrases such as rising prices, index, per capita income, inflation, stock market index, etc. in ordinary day-to-day language. There may be an exhaustive list to prove the case in favour of "mathematics for survival".

In addition to above speculation, the following points can be helpful in understanding the nature of mathematics.

1. **Mathematics has its own language and symbols**, which cut short the lengthy statements and help the expression of ideas or things by giving them exact form. For example, the following combination of signs and alphabets denotes a whole concept.

2. **Mathematics is the Science of logical reasoning** where results are developed through a process of reasoning. The results of theorems, laws and principals of mathematics are not upto discretion of the user. There is always a reason of step done in order to find a solution to a problem. If readers can go back in their childhood in school, your mathematics teacher would always have pursued to write reason at every point. In this connection, the words of Locke are worth mentioning here - "Mathematics is a way to settle in the mind - a habit of reasoning".

3. **Mathematics is also the science of inductive and deductive reasoning:** Inductive reasoning means when a particular property is true in a sufficient number of cases, then we can conclude that it will be true in all similar cases. Due to this nature of mathematics, mathematicians are of the view that "Mathematics in the making (beginning) is inductive and not deductive science". Likewise, mathematics is also deductive science and deductive reasoning is based on axioms, postulates self-evident truth, undefined terms and definitions. That is why, A.N. Whitehead has said rightly that, "Mathematics, in its widest sense, is the development of all types of deductive reasoning".

4. **Mathematics is an exact and precise science:** Mathematics is all about being exact and precise. There are no ambiguous statements or hidden assumptions. Definitions of concepts are precise and essential. If a problem is not well-defined with unique set of solutions, it is not a mathematical problem. In this connection, Courant etc. have expressed their views about the nature of mathematics in beautiful words as "Mathematics as an expression of the human mind reflects the active will, the contemplative reason, and the desire for aesthetic perfection. Its basic elements are logic and intuition, analysis and construction, generality - and individuality."

5. **Mathematics is universal:** Mathematical rules, laws and formulae are universal that can be verified at any place and time.

6. **Mathematics is of two types - Pure and Applied Mathematics:** Pure mathematics deals with the theories and principles without regard to their application to concrete things whereas Applied mathematics is the practical side of Pure mathematics. That is why, each and every invention and discovery in the field of physical, biological or social sciences owe much to applied "mathematics".

7. **Mathematics involves an intuitive method:** The first step in the learning of any mathematical subject is the development of intuition. This must come before rules are stated (or) formal operations are introduced. The teacher has to foster intuition in our young children, by following the right strategies of teaching.
   Intuition, when applied to Mathematics, involves the concretization of an idea not get stated in the form of some sort of operations (or) examples. Intuition is to anticipate what will happen next and what to do about it. It implies the act of grasping the meaning (or) significance (or) structure of a problem without explicit reliance on the analytic mode of thought. It is a form of mathematical activity which depends on the confidence in the applicability of the process rather than upon the importance of right answers all the time. It is up to the teacher to allow the child to use his natural and intuition way of thinking, by encouraging him to do so and honouring him when he does.

8. **Mathematics requires the application of rules and concept to new situations:** The study of Mathematics requires the learners to apply the skills acquired to new situations. The students can always verify the validity of mathematical rules and relationships by applying them to novel situations. Concepts and principles become more functional and meaningful only when they are related to actual practical applications. Such a practice will make the learning of Mathematics more meaningful and significant.

9. **Mathematics is an abstract science:** Mathematical concepts are abstract in the sense that they cannot be seen (or) felt in the physical world.
   For example:

i. Euclid's lines are supposed to have no width, and its points have no size. No such objects can be found in the physical world.
ii. Infinity is something that we can never experience and yet it is the central concept of Mathematics.
iii. Negative numbers do not correspond to any physical objects because we cannot have quantity less than nothing.

Hence, mathematical concepts cannot be learned through experiences with concrete objects. Some concepts can be learned only through their definitions and they may not have concrete counterparts to be abstracted from. Most of the mathematical concepts are such concepts without concretization and hence they are abstract. The concept of prime numbers, the concept of probability, the concept of a function, the concept of limits, concept of continuous functions, to list few are all abstract in the sense that they can be learned only through their definitions and it is not possible to provide concrete objects to correspond to such concepts. Even when concretization is

possible, they are only representation of the concepts and not physical object themselves.

10. **Mathematics is study of structures:** The dictionary meaning of 'structure' is "the formation, arrangement and articulation of parts in anything built up by nature or art". Therefore, a mathematical structure should be some sort of arrangement, formation (or) result of putting together of parts. For example, we take as the fundamental building units of a structure the members a, b, c,…..... of a non-empty set S, we hold together these building units by using one or more operations, namely addition or multiplication. A mathematical structure is a mathematical system with one or more explicitly recognized (mathematical) properties. We may create a structure from mathematical systems by making specific recognition of one or more of the commutative, associative or distribution properties that the system may have.

    With one or more basic structure at hand, one may construct other structures. Since plane analytic geometry is the study of subset or the Cartesian set Rex Re, where Re is the set of real numbers, plane analytic geometry may be considered as a superstructure based upon the structure known as real number system.

Thus, mathematics has got definite logical structure. These structures ensure the beauty and order of mathematics. Number systems, group, field, ring vector, space, etc. are all examples of mathematical structures.

Scope of Mathematics

Mathematics is all-pervasive. So, demarcation of its scope is a difficult task, if not impossible. It is science of all sciences and provides basis to all disciplines. By scope, it means the study area of a subject or discipline. In other words, scope refers to the questions or problems the subject or discipline is giving answer to. According to this definition, two main aspects of mathematics are Basic/Pure Mathematics and Applied Mathematics.

I. Basic/Pure Mathematics: The theoretical aspect of mathematics is termed as basic or pure mathematics. It involves systematic and deductive reasoning. It treats only theories and principles without regard to their application to concrete things. It consists of all those assertions as that if such and such proposition is true of that thing. It is developed on an abstract, self-contained basis without any regard to possible practical application that may follow.

The following are the sub-branches of pure mathematics:

- **Algebra:** It includes arithmetic, elementary and multivariate algebra, linear multivariate algebra, algebraic structure, etc.
- **Geometry:** It includes Euclidean and non-Euclidean geometry, projection, ANALYTIC geometry, trigonometry, differential and algebrical geometry.
- **Modern Mathematics:** It consists of the following topics:

a) **Set Theory -** Origin and definition, fundamental set concepts, Contorian set theory, postulates of axiomatic set theory, etc.

b) **Topology General** - topology, toplogical groups, differential topology, algebraic topology, etc.

c) **Algebraic Systems -** Groups, rings, field, vector space, etc.

- **Analysis:** It includes real and complex analyses, functional analyses, differential equation, Fourier theory of probability, vector and tensor.

II. Applied Mathematics: Applied Mathematics is the application of pure mathematics in developing the various means to serve the human and humanity. It considers those parts of mathematics theories that have certain direct or practical application to objects or actions in the material world. Principles of applied mathematics have been used to investigate phenomenon such as heat, light, electricity, sound, mechanics, astronomy, navigation, etc. Applied mathematics, thus, helps in solving the intricate problems of physical or real worlds. Like pure mathematics, it also has various sub-branches:

- Statistics
- Numerical Analyses
- Mathematical Theory of Optimization
- Automation Theory
- Information Theory
- Mathematical Aspects of Physical Theories: It includes:

i. Mechanics of Solids, Mechanics of Fluids, and Mechanics of Particles and Systems
ii. Quantum Mechanics
iii. Statistical Mechanic
iv. Electromagnetic Theory
v. Dimensional Analyses
vi. Theory of Relativity
vii. Riemannian Geometry

From above, many might consider the subject content of mathematics as complex and difficult. But it is a teacher's responsibility to create such teaching-learning environment, which addresses ways of acquiring and using knowledge, and are developed across the entire mathematics curriculum. They also can be applied across other content areas and real-world problems. The role of the teacher is to provide settings, models, and guidance for these processes to develop and to assess student skills in using these processes. The process standards are applied at every grade level and across all five content areas.

Engaging in mathematics is problem solving. Problem solving is what one does when a solution is not immediate. Students should build mathematical knowledge through problem solving, develop abilities in formulating and representing problems in various ways, apply a wide variety of problem-solving strategies, and monitor their mathematical thinking in solving problems. Problems become the context in which students develop mathematical

understandings, apply skills, and generalize learning. Students frequently solve problems in cooperative groups and even create their own problems.

Students should learn to reason and construct proofs as essential and powerful aspects of understanding and using mathematics. These processes involve making and investigating conjectures, developing and evaluating arguments, and applying various types of reasoning and methods of proof. Reasoning skills are critical for science, social studies, social skills, literature and most other areas of study.

Communication skills are an integral part of mathematics activities. Students must understand and use the language of mathematics in listening, speaking, reading and writing Mathematics communication involves specialized vocabulary and new symbol systems, and becomes a tool for organization and thinking. More than ever, students and teachers are "talking about math" with each other. Many new mathematics assessments require students to explain their thoughts and processes for solving problems in writing. Some mathematics teachers and mathematicians have tremendous understanding of mathematics concepts, yet have difficulty with communication skills. They cannot convey concepts on a level others will understand, or effectively use communication devices such as analogies and examples. Communication must be modelled with a full range of curriculum applications.

Making connections fosters deeper mathematics understanding and assists learning. Students are encouraged to make connections among different mathematics topics, across other content and skill areas, and into the "real" world. When introducing new concepts, it is critical that teachers assist students in making connections with previous, understood concepts. Linking prior knowledge results in more efficient and generalizable learning.

Students who are taught to make and apply representations across all mathematics topics have deeper insights of real world.

Practice Questions

1. "Mathematics is primarily taught on account of the mental training it affords, and only secondarily on account of the knowledge of facts it imparts." Discuss.

2. "Mathematics is the gateway, key and root of all sciences." Explain with examples.

3. "Mathematics is a language of scientific peoples." Explain.

4. Give conclusion about the meaning and definition of mathematics.

Aims and Objectives of Teaching Mathematics

Introduction

- The aims and objectives of mathematics teaching should be formulated on solid philosophical, sociological and scientific footings. The utility, appropriateness, practicability and timeliness of the objectives should be considered. It is important that objectives should be selected towards which the growth and development of an individual may be directed from a practical point of view.
- Objectives in any curriculum should be regarded as direction of growth and not as ultimate ends to be completely reached. Objectives strongly influence the organization of the curriculum, and at the same time, they provide the guidelines for the selection of teaching techniques. The aims and objectives of teaching a particular subject determine its place into curriculum in shaping the content and methods of teaching. The aims and objectives of mathematics teaching have undergone a tremendous change during the last few decades due to unexpected developments in the field of science, technology and mathematics in the twentieth century.

Aims of Mathematics Teaching

- If mathematics programme in the school is to be effective, we must know what we are trying to accomplish and then put in all our effort to achieve it, because aimlessness makes the work uninteresting and results in the wastage of time, energy and resources.
- Knowledge of aim of mathematics teaching in our schools can be drawn from the knowledge of values of mathematics teaching as aims and values are interdependent. Aims help in realization of values while the knowledge of values helps in setting the aim.

Aims of Mathematics Teaching

1. recognize that mathematics permeates the world around us
2. appreciate the usefulness, power and beauty of mathematics
3. enjoy mathematics, and develop patience and persistence when solving problems

4. understand and be able to use the language, symbols and notation of mathematics
5. develop mathematical curiosity and use inductive and deductive reasoning when solving problems.
6. become confident in using mathematics to analyze and solve problems both in schools and in real-life situations
7. develop the knowledge, skills and attitudes necessary to pursue further studies in mathematics
8. 8. develop abstract, logical and critical thinking and the ability to reflect critically upon their work and the work of others
9. develop a critical appreciation of the use of information and communication technology in mathematics
10. appreciate the international dimension of mathematics and its multicultural and historical perspective.

Objectives of Mathematics Teaching

Objectives of Mathematics teaching can be divided into the following categories.

A. Knowledge and Understanding

B. Investigating Patterns

C. Communication in Mathematics

D. Reflection in Mathematics

Knowledge and Understanding

Knowledge and understanding are fundamental to studying mathematics and form the base from which to explore concepts and develop problem-solving skills. Through knowledge and understanding, students develop mathematical reasoning to make deductions and solve problems. At the end of the course, the students should be able to:

1. know and demonstrate understanding of the concepts from the five branches of mathematics (number, algebra, geometry and trigonometry, statistics and probability, and discrete mathematics).

2. use appropriate mathematical concepts and skills to solve problems in both familiar and unfamiliar situations including those in real-life contexts.
3. select and apply general rules correctly to solve problems including those in real-life contexts.

Investigating Patterns

Investigating patterns allows students to experience the excitement and satisfaction of mathematical discovery. Mathematical inquiry encourages students to become risk-takers, inquirers and critical thinkers. The ability to inquire is valuable in the Mathematics and contributes to lifelong learning.

Through the use of mathematical investigations, students are given the opportunity to apply mathematical knowledge and problem-solving techniques to investigate problem, generate and/or analyze information, find relationships and patterns, describe these mathematically as general rules, and justify or prove them.

At the end of the course, when investigating problems, in both theoretical and real-life contexts, the students should be able to:

1. select and apply appropriate inquiry and mathematical problem-solving techniques
2. recognize patterns
3. describe patterns as relationships or general rules
4. draw conclusions consistent with findings
5. justify or prove mathematical relationships and general rules.

Communication in Mathematics

Mathematics provides a powerful and universal language. Students are expected to use mathematical language appropriately when communicating mathematical ideas, reasoning and findings- both orally and in writing.

At the end of the course, the students should be able to communicate mathematical ideas, reasoning and findings by being able to:

1. use appropriate mathematical language (notation, symbols, terminology, etc.) in both oral and written explanations.
2. use different forms of mathematical representation (formulae, diagrams, tables, charts, graphs and models)

3. move between different forms of representation.
4. Students are encouraged to choose and use ICT tools as appropriate and, where available, to enhance communication of their mathematical ideas. ICT tools can include graphic display calculators, screenshots, graphing, spreadsheets, databases, and drawing and Word processing software.

Reflection in Mathematics

Mathematics encourages students to reflect upon their findings and problem-solving processes. Students are encouraged to share their thinking with teachers and peers, and to examine different problem-solving strategies. Critical reflection in mathematics helps students gain insight into their strengths and weaknesses as learners and to appreciate the value of errors as powerful motivators to enhance learning and understanding. At the end of the course, the students should be able to:

1. explain whether their results make sense in the context of the problem
2. explain the importance of their findings
3. justify the degree of accuracy of their results where appropriate
4. suggest improvements to the method when necessary.

Bases for the Formulation of Objectives

A judicious formulation and selection of worthwhile objectives for any school subject goes a long way in enriching and shaping both the teaching and testing in that subject and such objectives should be evolved in relation to the needs of the individual in his society. The main sources for the formulation of objectives are:

1) The need and capabilities of the student
2) The demands of a student's social environment
3) The nature of subject-matter
4) The nature of education system
5) The availability of resources

The objectives should be formulated in the light of the above-mentioned factors. It is important that it is the need and demands of young generation which must be translated into educational objectives by identifying the new patterns of behaviour. The objectives thus formulated should be selected and classified. The criteria of good objective are:

6) Specific
7) Unambiguous
8) Useful
9) In accordance to general aims of education.

Bases for the Formulation of Objectives

The statement of objective is useful to the extent that it specifies what the learner must be able to do or perform when he is demonstrating his mastery of the subject. It is the task of the teacher to prepare clear and precise objectives of lesson before going into the classroom.

Benjamin Bloom et al. (1956) have made above-said task of teacher simpler by development of a taxonomy, which arranges educational objectives according to their characteristics and relationship with cognition, emotions and motor skills.

- Cognitive: Mental skills (knowledge)
- Affective: Growth in feelings or emotional areas (attitude or self)
- Psychomotor: Manual or physical skills (skills)

Within the domains, learning at higher levels is dependent on having attained pre-requisite knowledge and skills at lower levels, i.e., these domains further have levels of excellence to be achieved by learner. These are discusses in form of tabular form for concrete understanding as follows:

Cognitive Domain

The cognitive domain includes those educational objectives, which are related to the recall of knowledge, and the development of intellectual abilities and skills. If a student can comprehend the meaning of a piece of information to the point that he can grasp the relationship among them, restate them in his own words and take informed and intelligent actions, then he has developed his cognitive abilities.

The taxonomy of cognitive domain with six levels:

Level of Ability	Description of Ability	Action Verbs
1. Remembering	Recalling relevant knowledge from long-term memory. Recognition of terms, ideas, procedure, theories, diagram	Define, match, check, order, choose, find, recall, group, identify, label, list, select, locate, name, recognize, count, draw, find etc.
2. Understanding	Comprehension through comparing and contrasting. Students translate, interpret, extrapolate, but not see full implications or transfer to other situations, closer to literal translation.	Calculate, discuss, select, convert, restate, solve, classify, give examples, predict, review, describe, explain, identify, generalize, conclude, summarize, etc
3. Applying	Use of previous two levels in a new situation or unprompted use of abstraction. Apply abstractions, general principles or methods to specific concrete situations.	Relate, apply, change, choose, compute, demonstrate, discover, illustrate, predict, relate, show, sketch, solve, etc.
4. Analyzing	Separation of a complex idea into its constituent parts and an understanding of organization and relationship between the parts. Includes realizing the distinction between hypothesis and fact as well as between relevant and extraneous variables.	Analyze, calculate, categorize, compare, contrast, differentiate, discriminate, distinguish, examine, experiment, question, test, etc.
5. Evaluating	To make a judgment of ideas or methods using external evidence or self-selected criteria substantiated by observations or informed rationalizations.	Appraise, argue, assess, attach, choose compare, defend, estimate, judge, predict, rate, select, support, evaluate, etc.

6. Creating	Creative, mental construction of ideas and concepts from multiple sources to form complex ideas into a new, integrated and meaningful pattern subject to given constraints.	Arrange, collect, compose, construct, create, design, develop, formulate, organize, plan, prepare, propose, set up, etc.

Table 5.1: Bloom's Taxonomy of Educational Objectives for Cognition Level

In Table 5.1, first column is for level of ability which are in increasing order goes higher from remembering to creating; second column describes the level of ability in terms of mental processes involved, whereas third column is collection of action verbs of the level of ability. Action verbs are words which indicate a behavioral action of learner which can be observed or measured. Action verbs are the learner's artifact to be used as evidence of learning of the learner. In other words, by writing learning objectives using measurable verbs, teacher indicates explicitly what the student must do in order to demonstrate learning. For example, at remembering level, the fundamental cognitive process of brain is to revive information from memory. So, to demonstrate this level of ability, the learner will perform these measurable behavior, viz., recall, identify, list, define, locate, name, recognize, count, draw, find, etc.

Examples of Learning Objectives of Mathematics of Cognitive Domain

Students will be able to:

1. Identify the shape of triangle when showed.
2. Enlist the properties of rectangle.
3. Recall the statement of Pythagoras theorem.
4. Recall the formula for volume of cylinder.
5. Calculate the area of volume of cylinder.
6. Solve the questions related to volume of cylinder in real-life context.
7. Differentiate between type of equations.
8. Generalize the process of solving equations in two variables.

Affective Domain

Affective domain includes attitude, interests, values and apperception.

There are five categories of affective domain:

In Table 5.2, first column is for level of ability which are in increasing order goes higher from receiving to characterization; second column describes the level of ability in terms of emotional inclination processes involved, whereas third column is collection of action verbs of the level of ability. Here again, action verbs are to be used as evidence of affectional involvement of the learner in the classroom teaching-learning process of the subject.

Level of Ability	Description of Ability	Action Verbs
1. Receiving	Being aware and possessing a willingness to receive information or perspective. It runs along continuum with the learner starting as a passive participant to the learner directing or controlling the attention being given.	Asks, chooses, locates, points to, acknowledges, attentive, courteous, dutiful, follows, gives, listens, understands, etc.
2. Responding	The learners are no longer just giving their attention; they now take some form of action. It also falls along a continuum. Action occur without acceptance or understanding of necessity, voluntary action, and ends with a positive emotional element involved with the action.	Answers, assists, aids, complies, conforms, discusses, greets, helps, labels, performs, presents, tells, etc.
3. Valuing	Individual moves from assigning tentative yet consistent worth toward having conviction with certainty.	Appreciates, cherishes, treasures, demonstrates, initiates, invites, joins, justifies, proposes, respect, shares, etc.
4. Organization	As value becomes commitment, organization will need to take place. Attention is given to how commitments are connected and which take precedent.	Integrates, modifies, arranges, compares, combines, etc.
5. Characterization	Individuals will react to similar situations in a constant manner	Acts, discriminates, displays, influences, modifies,

by a value or value complex	based upon the values they have developed and nurtured.	performs, qualifies, questions, revises, serves, solves, verifies, etc.

Table 5.2: Taxonomy of Educational Objectives for Affective Level

Students will be able to:

Examples of General Objectives for Affective Domain of Mathematics Teaching- Learning:

1. Develop values such as discipline, efficiency, neatness and accuracy.
2. Develop sense of cooperation while working in groups.
3. Appreciate dignity of labour.
4. Develop interest in mathematics.

Examples of Specific objective using action verbs for affective domain of mathematics teaching and learning:

1. Attentive to classroom instructions.
2. Follow the classroom instructions.
3. Discuss the process of solving questions.
4. Take initiative in project or discussions.

Psychomotor Domain

Psychomotor objectives are those specific to discreet physical functions, reflex actions and interpretive movements. Traditionally, these types of objectives are concerned with the physically encoding of information, with movement and/or with activities where the gross and fine muscles are used for expressing or interpreting information or concepts. This area also refers to natural, autonomic responses or reflexes.

Terms in this area based on Anita Harrow's taxonomy:

Reflex Movements

Objectives at this level include reflexes that involve one segmented portion or reflexes of the spine and movements that may involve more than one segmented portion of the spine as intersegmental reflexes (e.g., involuntary muscle contraction). These movements are involuntary being either present at birth or emerging through maturation.

Fundamental Movements

Objectives in this area refer to skills or movements or behaviours related to walking, running, jumping, pushing, pulling and manipulating. They are often components for more complex actions.

Perceptual Abilities

Objectives in this area should address skills related to kinesthetic (bodily movements), visual, auditory, tactile (touch), or coordination abilities as they are related to the ability to take in information from the environment and react.

Physical Abilities

Objectives in this area should be related to endurance, flexibility, agility, strength, reaction-response time or dexterity.

Skilled Movements

Objectives in this area refer to skills and movements that must be learned for games, sports, dances, performances, or for the arts.

Non-discursive communication

Objectives in this area refer to expressive movements through posture, gestures, facial expressions, and/or creative movements like those in mime or ballet. These movements refer to interpretative movements that communicate meaning without the aid of verbal commands or help.

Level of Ability	Description of Ability	Action Verbs
1. Perception (awareness)	The ability to use sensory cues to guide motor activity. This ranges from sensory stimulation, through cue selection, to translation.	Chooses, describes, detects, differentiates, distinguishes, identifies, isolates, relates, selects, etc.
2. Set	Readiness to act. It includes mental, physical and emotional sets. These three sets are dispositions that predetermine a person's response to different situations (sometimes called mindsets).	Begins, displays, explains, moves, proceeds, reacts, shows, states, volunteers, etc..

3. Guided Response	The early stages in learning a complex skill that includes imitation and trial and error. Adequacy of performance is achieved by practicing.	Copies, traces, follows, react, reproduce, responds, etc.
4. Mechanism	This is the intermediate stage in learning a complex skill. Learned responses have become habitual and automatic.	Assembles, calibrates, constructs, dismantles, displays, fastens, fixes, grinds, heats, manipulates, measures, mends, mixes, organizes, sketches, etc.
5. Complex Overt Response	The movements can be performed with some confidence and proficiency.	Assembles, builds, calibrates, constructs, dismantles, displays, fastens, fixes, grinds, heats, manipulates, measures, mends, mixes, organizes, sketches, etc. NOTE: The Keywords are the same as Mechanism, but will have adverbs or adjectives that indicate that the performance is quicker, better, more accurate, etc.
6. Organization	The skilful performance of motor acts that involve complex movement patterns. Proficiency is indicated by a quick, accurate and highly coordinated performance, requiring a minimum of energy. This category includes performing without hesitation and automatic performance.	Arranges, builds, combines, composes, constructs, creates, designs, initiate, makes, originates, etc.
7. Adaptation	Skills are well developed and the individual can modify movement patterns to fit special requirements.	Adapts, alters, changes, rearranges, reorganizes, revises, varies, etc.

In Table 5.3, first column is for level of ability which are in increasing order goes higher from perception to adaptation; second column describes the level of

ability in terms of dexterity level in performing a pyscho-manual task, whereas third column is collection of action verbs of the level of ability.

Here again, action verbs are to be used as evidence of level of dexterity of the learner in the performing tasks related to the subject.

Examples of General Objectives for Affective Domain of Mathematics Teaching-Learning:

- Students will be able to:

1. Arrange objects according to their length.
2. Use measuring tape to measure dimensions of shapes.
3. Trace the diagrams.
4. Construct angle of 45° by using compass.

PRACTICE QUESTIONS

1. Explain the meaning and importance of educational objectives in mathematics teaching.
2. What should be the aims of teaching mathematics at school stage?
3. Explain appreciation and skill objectives of teaching mathematics at secondary stage.

NCF 2005: Objectives of Teaching Mathematics at Secondary and Higher Secondary Levels

Introduction to NCF 2005

The National Curriculum Framework (NCF) is one of four National Curriculum Frameworks published in 1975, 1988, 2000 and 2005 by the National Council of Educational Research and Training (NCERT) in India. The document provides the framework for making syllabi, textbooks and teaching practices within the school education programmes in India.

NCF 2005 has been translated into twenty-two languages and has its influence on school syllabi of seventeen states. The NCERT gave grants to each state to promote NCF in language of the state and to compare its current syllabus with the syllabus proposed, so that a plan for reforms could be made. Several states have taken up this challenge. This exercise is being carried out with the involvement of State Councils of Education Research and Training (SCERT) and District Institutes of Education and Training (DIET).

The Document is divided into five areas:

- Perspective
- Learning and Knowledge
- Curriculum Areas, School Stages and Assessment
- School and Classroom Environment
- Systemic Reforms

Emergence of NCF 2005

As per the directions of the Human Resource Development Minister, the NCERT took up the assignment of reviewing the National Curriculum Framework for School Education in the light of the report Learning without Burden.

A National Steering Committee under the Chairmanship of Professor Shri Yash Pal formed 21 National focus groups. Members of these committees included representatives of institutions of advanced learning, NCERT's own faculty, school teachers and non-governmental organizations. Deliberations at National and state level, and public opinions were invited by giving wide advertisements.

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The NCF 2005 begins with a quotation from Tagore's essay Civilization and Progress in which the poet reminds us that a 'creative spirit' and 'generous joy' are key in childhood, both of which can be distorted by an unthinking adult world. Seeking guidance from the constitutional vision of India as a secular, egalitarian and pluralistic society, founded on the values of social justice and equality, certain broad aims of education have been identified in this document NCF 2005. These include independence of thought and action, sensitivity to others' well-being and feelings, learning to respond to new situations in a flexible and creative manner, predisposition towards participation in democratic process, and the ability to work towards and contribute to economic processes and social change. For teaching to serve as a means of strengthening our democratic way of life, it must respond to the presence of first generation schoolgoers, whose retention is imperative owing to the constitutional amendment that has made elementary education a fundamental right of every child. The fact that learning has become a source of burden and stress on children and their parents is an evidence of a deep distortion in educational aims and quality.

Guiding principles of curriculum development (as prescribed as NCF 2005) To correct above speculated distortions in current curriculum framework and to correct these distortions, the present NCF proposes five guiding principles for curriculum development:

1. Connecting knowledge to life outside the school.
2. Ensuring that learning shifts away from rote methods.
3. Enriching the curriculum so that it goes beyond textbooks.
4. Making examinations more flexible and integrating them with classroom life.
5. Nurturing an overriding identity informed by caring concerns within the democratic polity of the country.

The NCF 2005 was framed keeping the above-mentioned guiding principles as to implement many good ideas that have already been articulated in the past.

Chapterization in NCF 2005

The document of NCF has 5 Chapters plus Examination Reforms dealing with different aspects of curriculum.

Chapter – 1

- Strengthening a national system of education in a pluralistic society.
- Reducing the curriculum load based on insights provided in ‘ Learning without Burden’.
- Systemic changes in tune with curricular reforms.
- Curricular practices based on the values enshrined in the constitution, such as social justice, equality and secularism.
- Ensuring quality education for all.
- Building a citizenry committed to democratic practices, values, sensitivity towards gender justice, problems faced by the scheduled Castes and the Scheduled Tribes, needs of the disabled, and capacities to participate in economic and political processes.

Chapter – 2

- Reorientation in our perception of learners and learning.
- Holistic approach in the treatment of learners' development and learning.
- Meeting learning disability needs through data-based and need-specific programmes.
- Learner engagement for construction of knowledge and fostering creativity.
- Active learning through experiential mode.
- Adequate room for voicing children's thinking, curiosity and questions in curricular practices.
- Connecting knowledge across disciplinary boundaries to provide a broader frame for insightful construction of knowledge.
- Forms of learner engagement - observing, exploring, discovering, analyzing, critical reflection, etc. are as important as the content of knowledge.
- Activities for developing critical perspectives on socio-cultural realities need to find space in curricular practices.

- Local knowledge and children's experiences are essential components of textbooks and pedagogic practices.
- School years are a period of rapid development with changes and shifts in capabilities, attitudes and interests that have implications for choosing and organizing the content and process of knowledge.

Chapter – 3

Chapter 3 elaborates the prescription for Nine different school subjects:

- ✓ Language
- ✓ Mathematics
- ✓ Science
- ✓ Social Sciences
- ✓ Work
- ✓ Health and Physical Education
- ✓ Work and Education
- ✓ Education for Peace
- ✓ Habitat and Learning

Health and physical education are necessary for the overall development of learners.

Through health and physical education programmes (including yoga), it may be possible to handle successfully the issues of enrolment, retention and completion of school.

Chapter – 4

- Availability of minimum infrastructure and material facilities, and support for planning a flexible daily schedule are critical for improved teacher performance.
- A school culture that nurtures children’s identities as learners’ enhances the potential and interests of each child.
- Specific activities ensuring participation of all children – able and disabled – are essential conditions for learning by all.

- The value of self-discipline among learners through democratic functioning is as relevant as ever.
- Participation of community members in sharing knowledge and experience in a subject area helps in forging a partnership between school and community.
- Reconceptualization of learning resources in terms of:

- Textbooks focused on elaboration of concepts, activities, problems and exercises encouraging reflective thinking and group work.

- Supplementary books, workbooks, teachers' handbooks, etc. based on fresh thinking and new perspectives.

- Multimedia and ICT as sources for two-way interaction rather than one-way reception.
- School library as an intellectual space for teachers, learners and members of the community to deepen their knowledge and connect with the wider world.
- Decentralized planning of school calendar and daily schedule and autonomy for teacher professionalism practices are basic to creating a learning environment.

Chapter – 5

- Quality concern, a key feature of systemic reform, implies the system's capacity to reform itself by enhancing its ability to remedy its own weaknesses and to develop new capabilities.
- A broad framework for planning upwards, beginning with schools for identifying focuses areas and subsequent consolidation at the cluster and block levels, could form a decentralized planning strategy at the district level.
- Meaningful academic planning has to be done in a participatory manner by Headmasters and teachers.
- Monitoring quality must be seen as a process of sustaining interaction with individual schools in terms of teaching-learning processes.
- Professional training of teachers can be strengthened by linking it to:

- Post-graduate studies in different subjects.

- Provisions for integrated undergraduate studies in teacher education.
- Inclusion of a course on language proficiency as an integral component.

- Engaging the trained with the larger context of education, interacting with children in real contexts and critically questioning their own beliefs about knowledge and learning, gender, caste, equity and justice.
- Shifting the focus from pure disciplinary knowledge to the learner and his/her context.
- In-service education needs to become a catalyst for change in school practices.
- Panchayat Raj system should be strengthened by evolving a mechanism to regulate the functioning of parallel bodies at the village level so that democratic participation in development can be realized.

Examination reforms

Reducing stress and enhancing success in examination necessitate:

- Shift from content-based testing to problem solving and understanding. For this to happen, the present typology of the question paper must change.
- Shift toward shorter examinations.
- Setting up of a single nodal agency for coordinating the design and conduct of entrance examinations.
- Availability of multiple textbooks to widen teachers' choices and provide for the diversity in children's needs and interests.
- Sharing of teaching experiences and diverse classroom practices to generate new ideas, and facilitate innovation and experimentation.
- Development of syllabi, textbooks and teaching-learning resources could be carried out in a decentralized and participatory manner involving teachers, experts from universities, NGOs and teachers' organizations.

Mathematics education from perspective of NCF 2005

"The essence of mathematics resides in its freedom."

-George Cantor

The above quote by George Cantor is the essence of whole mathematics. When learner is provided with freedom to learn, i.e., choosing the methods that suits his intellect is the jest of joyful learning for mathematics. But, during the course of time, the essence of the subject started getting lost. Mathematics became a subject to reckon with formulae, theorems and proofs, with little application in everyday life.

Thus, Mathematics leads to a common dear in students: Math-phobia.

Issues and challenges of school mathematics from perspective of NCF

The National Curriculum Framework discusses these issues and issues relating to the teaching of mathematics in school comprehensively.

Issues in mathematics education

1. A sense of fear and failure regarding Mathematics among a majority of children.
2. A curriculum that disappoints both a talented minority as well as the non-participating majority at the same time because it offers no challenges.
3. Methods of assessment that encourage the perception of Mathematics as mechanical computation - problems, exercises and methods of evaluation are mechanical and repetitive with too much emphasis on computation.
4. Lack of teacher preparation, confidence and support in the teaching of Mathematics.

The challenge of mathematics in schools

As per NCF 2005, the main goal of mathematics education is the development of children's ability of mathematization.

"It is more useful to know how to Mathematize than to know a lot of Mathematics."

- David Wheeler

What is mathematization?

To be able to mathematize is as simple as being able to be well versed in the language of mathematics and make meaning of it. It primarily consists of:

1. Forming equations
2. Representations

3. Choosing variables
4. Arriving at a conclusion logically

Knowing a lot of Mathematics implies being able to do a variety of computations according to a set procedure that is learnt. Mathematization, on the other hand, involves the ability to apply Mathematics according to the need of the situation - to be able to think mathematically.

Examples:

1. Length of a rectangular field is two times its width and its area is 400 square meters. This situation can be expressed (mathematized) as $2x.x = 400$ choosing x as a variable representing width of the field. But it is important that the context of the problem is relevant to the child.
2. In a game of lacrosse, Oklahoma beat Texas by 10 points. The total points won in the game is 50. What are the points scored by Texas?

This context would be irrelevant to a child in India who would know nothing about lacrosse.

But something like:

Rohit keeps a record of his daily expenses. He spent 10 rupees more on fruits than on vegetables. In all, he spent 50 rupees. How much did he spend on fruits?

Both questions address the same concept, but the context is critical.

Aims of school mathematics according to NCF 2005

The document of NCF 2005 can be summarized into two main aims of mathematics education: one is narrow aim and the other is much broader and much needed for achievement of former.

Narrow aim:

- To develop numeracy skills like decimals, percentages, measurements, etc.

Higher aim:

- To create mathematical learning environments, where processes like formal problem solving, use of heuristics, estimation and approximation, optimization, use of patterns, visualization, reasoning and proof, making connections and mathematical communication take precedence.

The need of the hour is to shift emphasis from narrow aim to higher aim.

Now, we will be discussing highlighted terms in higher aim statement of Mathematics.

1. Estimation refers to judgement of the worth, value, quantity or extent of something:
2. Visualization
3. Patterns
4. Reasoning and Proof

1. Estimation refers to judgement of the worth, value, quantity or extent of something:

It could be as simple as estimating how much water and milk is required to make a cup of tea and inducting it to 2 cups and so on. Or it could be utilizing learnt concept of area of a rectangle, to estimates the area of the classroom. After the concept of surface area of a cuboid, the area to be painted in a room could be estimated.

2. visualization:

Mathematics, unlike science, is an abstract concept. The concept of numbers, lines, points, etc. are all abstract. Consciously or subconsciously, we associate objects with numbers or geometrical shapes in our minds.

For example: 1 + 1 = 2 could be equivalent to adding 1 pencil with another pencil. An abstract concept can be made to understand using physical objects.

3. patterns:

A pattern constitutes a set of numbers or objects in which all the members are related with each other by a specific rule.

For example: What is sum of 1+2+3+4+5+... + 100?

Adding the first number and last, second and second last, third and third last and so on:

1+100-101

2+99-101

3+98 =101

50+51=101

Therefore, 1+2+3+...+100=101×50=5050

If we do the same for. 1+2+3+ .... + 101, we get

1 + 101 = 102

2+100 = 102

50+52102..

51+51=102

Therefore, 1+2+3+...+101 102×515202

Following this pattern, we arrive at the formula that we have all been using:

Sum of 1st n natural numbers = (n)*(n+1)/2

The NCF primarily focuses on constructing of knowledge by a child. In the context of math, this means that the child's ability to come up with a formula is more important than being able to correctly use well-known formulae.

4. Reasoning and proof:

The NCF says that a child should learn to evaluate arguments, and make and investigate conjectures and understand various methods of reasoning.

We have been taught that 0/0 is undefined! Why? Because, we cannot determine on one fixed value for quotient. That is why, it is undefined. But mostly, we miss out such simple reasoning in search of lengthy and complicated mathematical proofs. Giving valid alternative proofs lead us to multiple approaches to solve a single problem.

NCF 2005 EMPHASIZES MULTIPLICITY OF APPROACHES

Very often, there is more than one way of solving a problem. Exposing students to more than one approach gives them a choice to work with the best one that suits them and also given them an option to verify their answers.

Multiplicity of approaches is crucial for liberating school mathematics from the tyranny of the one right answer , found by applying the one algorithm taught. When many ways are available, one can compare them, and decide which is appropriate when, and in the process gains insight.

For example: What are the different ways of obtaining 10 through arithmetic operations?

10=5*2

10=100/10

10=15-5 and so on.

Organization of mathematics school curriculum according to NCF 2005

Pre-primary (nursery-kg):

According to NCF 2005, mathematics curriculum at primary stage should be based on the following principles:

1. Learning occurs through play rather than through didactic communication.
2. Rather than the rote learning of number sequence, children need to learn and understand, in the context of small sets, the connection between word games and counting, and between counting and quantity.
3. Making simple comparisons and classifications along one dimension at a time, and identifying shapes and symmetries, are appropriate skills to acquire at this stage.
4. Encouraging children to use language to freely express one's thoughts and emotions, rather than in predetermined ways, is extremely important at this and at later stages.

Primary (i-v):

NCF 2005 recommends the following basics to arrange curriculum of mathematics at primary stage of school:

1. Having children develop a positive attitude towards, and a liking for Mathematics at the primary stage is as important as developing cognitive skills and concepts.
2. Mathematical games, puzzles and stories help in developing a positive attitude and in making connections between Mathematics and everyday thinking.
3. Due importance must be given to shapes, spatial understanding, patterns, measurement, and data handling and concepts from abstract to concrete.

Upper primary (vi-viii):

According to NCF 2005, mathematics curriculum at upper primary stage should be based on the following principles:

1. Here, students get the first taste of the application of powerful abstract concepts that compress previous learning and experience. This enables them to revisit and consolidate basic concepts and skills learnt at the primary stage, which is essential from the point of view of achieving universal mathematical literacy.
2. Students are introduced to algebraic notation and its use in solving problems and in generalization, to the systematic study of space and shapes, and for consolidating their knowledge of measurement.
3. Data handling, representation and interpretation from a significant part of the ability to deal with information in general, which is an essential 'life skill'.

The learning at this stage also offers an opportunity to enrich students spatial reasoning and visualization skills.

Secondary (ix-x):

According to NCF 2005, mathematics curriculum at secondary stage should be based on the following principles:

1. Students now begin to perceive the structure of Mathematics as a discipline. They become familiar with the characteristics of mathematical communication: carefully defined terms and concepts, the use of symbols to represent them, precisely stated propositions and proofs justifying propositions. These aspects are developed particularly in the area of geometry.
2. Students develop their facility with algebra, which is important not only in the application of mathematics but also justification and proofs.
3. Students integrate the many concepts and skills that they have learnt into a problem- solving ability. Individual and group exploration of connections and patterns, visualization and generalization, making and proving conjectures are important at this stage can be encouraged through the use of appropriate tools that include concrete models as in Mathematics laboratories and computers.

Higher secondary (xi-xii):

According to NCF 2005, mathematics curriculum at primary stage should be based on the following principles:

1. To provide students with an appreciation of the wide variety of the application of Mathematics, and equip them with the basic tools that enable such application.
2. A careful choice between the often-conflicting demands of depth versus breadth needs to be made at this stage. The rapid explosion of Mathematics as a discipline, and of its range of application, favours an increase in the breadth of coverage.
3. Such increase must be dictated by mathematical considerations of the importance of topics to be included. Topics that are more naturally the province of other disciplines may be left out of the Mathematics curriculum. The treatment of topics must have an objective, i.e., the communication of mathematical insights and concepts, which naturally arouse the interest and curiosity of students.

PRACTICE QUESTIONS

1. Explain difference between Narrow and Higher aim of Mathematics Education as explained in NCF 2005.
2. Give recommendations by NCF 2005 for Mathematics Education at upper primary and secondary stage of school.
3. What are the guiding principles of curriculum development as given by NCF 2005?

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Values of Teaching Mathematics

Introduction

There is a popular belief about the subject mathematics that it is a neutral subject, which is true. Western countries teach mathematics in a neutral way without attaching any value to it. But many Asian countries teach mathematics by linking it to different values. Values of mathematics are mostly implicit and cannot be directly imposed on the learners. These values are embedded in the nature of content and process of content delivery. Also, these values vary according to the teacher delivering the content. This chapter will discuss these values in detail.

Values or Importance of Mathematics Teaching

Mathematics is a vital component of school education. That is why, it has become a compulsory subject in the school curriculum. But some times, the questions arise - Why should this subject be taught to everyone? What is the utility, value or importance of this subject in the life of the individual? What is the place of this subject in the school curriculum? What are the educational values of this subject? Knowledge about the values of the, subject will help the teacher and students to avoid a lot of meaningless efforts and will provide a base for the proper evaluation of teaching of this subject. Due to following educational and other multi-factious values to the individual as well as to the society, this subject is taught to the students in the schools.

1. **Practical or utilitarian value:**

   Right from morning till night, all our activities and engagements are controlled and fashioned, directly or indirectly by mathematics. Therefore, one need not emphasize upon the everyday use of certain amount of mathematics by every man, rich or poor. Even the unskilled labourers or small shopkeepers have got to calculate their wages, earning and busy things in the bazaar. They can get on simple mathematical calculation very well without learning how to read and write.

   Take any example from the social life of the country. Accountancy, banking, engineering, teaching, computer, life insurance, insurance agents, railways and postal jobs are such professions which depend on the sound knowledge of mathematics.

   In order to avoid confusion and chaos in the present complex society, knowledge of mathematics helps in all these world's entire commercial system for fixing timings, ratio, prices, wages, rates, ration, fares, percentages, interests, exchanges, commissions, discount, units of length, breadth, area, volumes, etc.

   Mathematics is the base of all essential knowledge and progress in science and technology. Bacon is right when he says, "Mathematics is the gate and key of all sciences". Whatever comforts science has given us, that are all due to the knowledge of mathematics.

Knowledge of mathematics is also useful in preparing budgets for family, business, firm, school, college, university, state and nation. While making purchases from the bazaar, arranging a party, admitting the child in the school or college, selling or buying a property, celebration of marriages, going to some hill station in vacations, building a house, mathematical considerations, etc. are always kept in mind.

Any person ignorant of mathematics will be at the mercy of others and will be easily cheated. Sometimes, the person is under debt or sometimes well of temporarily and spends blindly due to lack of sense of calculation and art of economy. Bacon has rightly said, "Neglect of mathematics work injury to all knowledge since, he who is ignorant of it, cannot know other sciences or the things of the world. And, what is worse, men who are thus ignorant are unable to perceive their own ignorance and so do not seek a remedy".

Mathematics has enabled people to use their leisure to the best advantages. Even the rising or setting of the sun appearing of stars and moon, change of season, rotation of planets, etc. in the nature also follows mathematical principles.

Thus, practical or utilitarian value of mathematics is not emphasizing here because it is directly useful to the man in the street in his everyday life, but because mathematics is useful to man in better understanding of the world around him and to have a more rational mode of life and because it is useful to him leading an intelligent and well-balanced civilized life.

2. **Disciplinary value:**

Mathematician Schulze has rightly stated, "If mathematics, however, had no disciplinary value its teaching in the secondary schools could hardly be justified solely on ground of its bread-and-butter value". Students, while learning mathematics, learn reasoning. If properly taught, mathematics discourages memory on the parts of the students. The student of mathematics does not take decisions on their discretion but tries to apply logic, reasoning and thinking power, and can very well discriminate what is good and what is bad.

The reasoning in mathematics possesses certain values that make it specially fitted for training the minds of the pupils. These values are: (a) accuracy, (b) certainty, (c) originality and (d) verification.

a) **Accuracy:** While learning mathematics, the students must think accurately to arrive at desired result. In other subjects, the students may hide their ignorance by mere juggling of words without understanding, but in mathematics, they cannot do such things. In this way, the student learns the value of appreciation of accuracy and makes it a principle of their life.

b) **Certainty:** In mathematics, as the answer is either right or wrong and there is no possibility of disagreement of the teacher with the student, the student develops the habit of being certain about his work.

c) **Originality:** Student of mathematics cannot do well without original thinking and intelligent reasoning, as solution to most of the problems is the result of original

thinking. This habit of originality helps the student to face the daily life problems with confidence.

d) **Verifiability:** In mathematics, the solutions can be verified by applying reverse method. For example, if we wish to check our quotient in divisibility is valid, we just multiply quotient and divider and add quotient if we get value equal to dividend, our answer is correct. Hence, the verifiability of solution in mathematics students develops the habit of self-evaluation, achievement and confidence among the students.

Thus, from above, it is clear that habits of simplicity and clearness, accuracy, certainly in expression, originality in thinking and verification of results are formed and strengthened by the study of this subject. Apart from above, mathematics develops certain other good habits among the students such as power of concentration, habit of hard work, regularity, punctuality, neatness, cleanliness and orderliness of work by its nature.

3. **Intellectual value:**

The great value of teaching mathematics is that it introduces us to new ways of thinking and reasoning. It sharpens the intellect, and makes learner more careful and systematic in their reasoning. It inculcates a spirit of enquiry, the capacity to know the unknown, and the strength to face hardship and failures. The pursuit of mathematics requires diligence and patience. Hubsch has rightly said, "Mathematics sharpens the minds of the people in the same way as some stone sharpens the tools."

The process of problem solving in mathematics boosts the development of intellectual or mental power. After understanding the problem and what is to be found out, all concerning facts and techniques for the solution of the problem are carefully analyzed to choose the most relevant ones and then to reach at some conclusion. In this way, mathematics trains the minds of the people and helps to understand, evaluate, and solve numerous mathematical and social problems of life. Thus, the individual begins to reason things he meets in daily life, discharges his duties in a better manner, and leads a happy and successful life in the society.

Mathematics develops the ability to perform necessary computations with accuracy and reasonable speed. It also develops an understanding of process of measurement and of the skill needed in the use of instrument of precision. National Policy on Education (1986) lays down the importance of mathematics as a vehicle for developing creativity. In other words, mathematics helps in the development of intellectual power like power of imagination, observation, originality, creativity, and systematic thinking and reasoning.

4. **Psychological value:**

The teaching and learning process of mathematics is based on fundamental principles of learning by doing, learning by observation, from concrete to abstract in which needs, interests, ability and motives of the child are fully taken into account.

Moreover, mathematics satisfies common instincts as creativeness, self-assertion and curiosity. The interests innate tendencies and aptitudes of the student are best utilized.

5. Cultural value:

The term culture means the mode of living of people. Mathematics has played an important role in determining the culture and civilization of a country from time to time. It has affected our way of thinking and way of living. The prosperity of man and his cultural advancement are considerably dependent upon the advancement of mathematics. It is due contribution of mathematical knowledge to the progress of various vocations such as engineering, medicine, industry, railways, roads, computers, building, agriculture and economic system, that we are living in such an advance culture.

Mathematics is called the backbone of our civilization and cultural advancement because a country's civilization and culture is reflected in the knowledge of mathematics it possesses. For example, James Watt, inventor of steam engine; Galileo, the inventor of telescope; Newton, the discoverer of law of gravity; Graham Bell' the inventor of telephone; Baird, the inventor of television were all great mathematicians. And today also, it is mathematics which helps in preparing maps with the help of which trade and travelling become possible; navigators can know the way in the sea; modern machines, telephone, telegraph, railways, cinema, electricity, television and video films which we see today. In all these things, knowledge of mathematics is utilized.

6. Aesthetic value:

There is a popular misconception about nature of mathematics that mathematics is dull, dry, difficult and uninteresting. This fallacy can be attributed to their unwillingness to learn or they might have teacher who tough them in a dull and improper way. Otherwise, every mathematician appreciates the aesthetic aspect of his discoveries and inventions. Learner might remember intrinsic charm and pleasure they might have felt after solving some mathematical problem.

Student of mathematics can understand, appreciate and relish any piece of arts, drawing, painting, architecture, music or dance or poetry in a better way than any other person who does not have mathematical knowledge. Regularity, symmetry, order or arrangement in any poetry, arts or drawing is nothing but mathematics. Similarly, all musical instruments are played on the set rule of mathematics. That is why, mathematician Leibnitz has said, "Music is a modern hidden exercise in arithmetic of mind unconscious of dealing with numbers".

Not just looking at beautiful piece of art require mathematical sense, but the creation of an art piece also requires the same conscious or sub-conscious sense of mathematics, e.g., the art of beautiful landscape, silver or gold ornament, designer dresses, decoration of room, flowers in flowering pot, etc. lie in the hands of the arrangement made with the help of mathematics.

7. **Moral value:**

Mathematics develops morality by teaching truthfulness, reasoning, honesty, patience, self-control, self-confidence, respect for other's opinions, and discrimination between good and bad. In mathematics, right is right, true is true and wrong is wrong. The remarks of Greek philosopher Dutton may be worth mentioning here, "Mathematics does furnish the power for deliberate thought and accurate statement, and to speak the truth whereas gossip, flattery, slander and deceit, all speak from a slovenly mind that has not been trained by mathematics".

8. **Social value:**

Mathematics has also got social value. History of mathematics tells us that whenever a country has given due weightage to the teaching of mathematics, it has made a tremendous progress. In this connection, Napoleon has rightly said that, "The progress and improvement of mathematics are linked to the prosperity of the state".

Mathematical facts and processes are useful and necessary for the proper understanding of the progress of society and civilization in a variety of ways. Administrative reports and budge speeches are always full of graphs, tables and financial forecast involving compound interest, law and geometrical progression. Mathematics has also raised the living standard of the people and makes the life of the people more comfortable.

Mathematics has helped in bringing together the countries of the world, which are separated from each other physically. Mathematics helped man to discover the mysteries of nature, and to overcome superstitions and ignorance. Hence, teaching of mathematics is inevitable in our schools.

9. **Vocational value:**

The study of mathematics is helpful in a number of professions. It forms the basis of so many studies, which are purely vocational in nature. For example, a student of mathematics can choose medical, engineering, technical profession, teaching, agriculture, banking, computer engineer, or some statistical profession in which he is interested in and fit for. The study of mathematics at school forms the basis of many useful hobbies and other productive activities in the later life of the student.

10. **International value:**

All mathematicians, irrespective of their caste, colour or creed, have contributed towards the progress of mathematics. Any new idea brought in the field of mathematics does not take much time to become an international property. Mathematicians of the whole world cooperate with each other. In spite of the fact that they belong to different countries, mathematics books and research journals of teachers, mathematicians and researchers from one country of the world; and exchange of teachers, mathematicians and researchers from one country to another country helps in achieving international understanding, brotherhood and peace.

Unit 2 (C): Values of Teaching Mathematics

Practice Questions

1. Discuss the educational values of mathematics by making special reference to its place in life.
2. "From a labourer to a educationist, everybody uses mathematics." Justify.
3. "Knowledge of Mathematics is essential for creation of Art." Comment.

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Maxims of Mathematics Teaching

Introduction

The role of the teacher is considered main in the classroom. He is responsible for creating such an ambience as gives rise to maximum learning activities and various experiences may be achieved. But, practically, it is not as convenient as it looks. However, talented, scholarly and able a teacher may be, he would be considered unsuccessful if he or she is unable to transfer learning to students. Teaching is an art which encompasses the following:

- Mastery over subject
- Scientific knowledge of teaching style for transfer of knowledge to pupils

In order to achieve this objective, teacher's subject knowledge is not the only factor which is required. In addition to this, the teacher should know some maxims with the help of which the teacher may present the subject-matter before the students effectively as well as efficiently.

Meaning of Maxims

Maxims have been formulated by the psychologists, educationists, pedagogues and preceptors on the basis of their experiences. These maxims are reliable and universally applicable. Different maxims of teaching are applied in different teaching situations on the basis of well-planned strategy and logic. Maxims of teaching act as the springboard which catalyzes the momentum of teaching-learning process, and thus helps the teacher in achieving the pre-determined objectives of education besides providing contentment to both the learners as well as the teacher.

The meaning of maxims of teaching is very simple. Those general ideas and methods of doing the work which prove helpful in the task of teaching are termed as maxims of teaching. Maxims of Teaching are the universal facts found out by the teacher on the basis of experience. They are of universal significance and trustworthy. The knowledge of different maxims helps the teacher to proceed systematically. It also helps to find out his way of teaching, especially in the early stages of teaching.

Maxims of Mathematics Teaching

Following are the most commonly used maxims in teaching of maxims.

1. **From known to unknown:**

This maxim is based on the assumption that the student knows something, We are to increase his knowledge and widen his outlook. We have to interpret all new knowledge in terms of the old. It is said that old knowledge serves as a hook on which the new one can be hung. Known is trustworthy and unknown cannot be trusted. So, while teaching, we should proceed from known and go towards unknown. For example, while teaching any lesson, the teacher can link the previous experiences of the child with the new lesson that is to be taught. When a child enters into school, he possesses some knowledge and it is

the duty of teacher to enlarge his previous knowledge. Whatever he possesses should be linked with the new knowledge. If we link new knowledge with the old knowledge, our teaching becomes clearer and more definite. This maxim facilitates the learning process and economizes the efforts of the teacher and the taught. For example, to teach logarithm, the teacher must use previous knowledge of process of indices.

2. **From Simple to Complex:**

Learning is the main objective of a teacher. During the teaching-learning process, the teacher must proceed from simple concepts to complex concepts. Teaching of simpler content first makes the learner motivated, interested and encouraged to move ahead towards attempting difficult content. Once students exhibit interest in learning, the teacher introduces complex content in gradual manner. On the other hand, if complex matter is presented first, the learner becomes upset, feels bored) and finds himself in a challenging situation. For example, concept of Simple Interest is taught before the concept of compound interest.

3. **From Particular to General:**

While teaching, the teacher should first of all take particular statements, and then on the basis of those particular cases, generalization should be made. General facts, principles and ideas are difficult to understand and the teacher should always first present particular things and then lead to general things, According to this maxim, the teacher should present some specific examples before pupil. Then the same example should be evaluated, and after understanding the fact material. pupils should be motivated to derive general principles.

For Example,

- $2 \times 2 \times 2 = 23$; $4 \times 4 \times 4 \times 4 = 44$ (Particular)
- $a \times a \times a = a^3$ (general)

4. **From Concrete to Abstract:**

The mental development of the pupil begins with the concrete objects, and afterwards, he gains micro-words for them. Therefore, to begin the education of pupils, the concrete object and fact should be made known first.

For example,

- Addition/subtraction/multiplication/division of objects like apples, books, pencils, etc.
- Addition/subtraction/multiplication/division of variables x, y, z.

5. **From Whole to Part:**

In this approach, the whole of the concept is taken for study first and then its various sub-divisions are explained. This makes the education scientific.

The knowledge acquired in this way is more stable. The teacher should present before the pupils the new teaching matter as a whole, and in an organized way first and then its parts should be explained on the basis of this 'whole' and organized teaching matter.

For example,

- Integers (whole)
- Rational and irrational numbers (part)

Benefits of Using Maxims

- Simplify the process of teaching
- Joyful teaching and learning environment
- Purposeful teaching
- Create creativity among students
- Analysis and synthesis by students
- Develop scientific attitude
- Learning by doing
- Develop critical and logical thinking in Mathematics.

Practice Questions

1. Elaborate the need of using concrete to abstract maxim in Mathematics Teaching.
2. "Moving from known to unknown is natural flow of learning." Comment.

Approaches of Curriculum Construction

Introduction

Curriculum is intimately related with all aspects of education. While education is a developmental process towards a converted goal, curriculum is the input goal-oriented direction to that process Curriculum is the plan for guiding the goal-oriented educative process

After topics have been selected according to the relevant fundamental principles described above, they have to be systematically arranged so as to facilitate meaning full and effective transaction. The content should be arranged in a systematic manner. In order to realize the objective, it is inevitable to organize the curriculum in the most psychological and logically coherent manner. There are different approaches for organizing the curriculum. They are spiral, topical, concentric and an unit approach.

Different Approaches

A discussion on the general approaches being adopted for organizing curriculum was made earlier. The approaches are:

(a) Concentric and Spiral approach

(b) Topical approach

The Concentric and Spiral Approach

The concentric approach is also known as spiral approach. In this, the whole curriculum is spread over a number of years. A general treatment of almost all the topics are attempted at the beginning and it is developed in successive years according to the mental development of the pupils. In the beginning of the course, the whole aspect is given to pupils in a simplified way. In the next year, more and more details of its parts are added. It follows the maximum of teaching, such as from whole to part, simple to complex, easy to difficult, etc. Among educationist of modern times, Burner is the main exponent of the approach is maintained. Sometimes, this approach is referred to as concentric approach. But the term "spiral approach" is preferred to the other. The term spiral gives the additional implication that while attempting gradation, the linkage too is taken care of and the continuing of the topic concerned is never broken) While conceiving it as concentric, only the widening of the scope is indicated, but the linkage is not taken care of.

The concentric approach is nothing, but devising a strategy that fosters continuous unbroken learning of the subject-matter. According to this approach, children in a primary classes begin to develop simple generalization about man carrying of his everyday activities. They work with more and more complex items of information, and as a result, deepen and reshape the dimension of the related generalization already developed earlier. For example, it is very important that children should know about the concept of addition, multiplication and division in the primary classes. The information about this unit will be imparted through word problems. In the middle stage, the information will be imported through performing addition, multiplication, etc. of variables in an equation. In the secondary stage, the pupils still learn to construct equations in variables and constants.

Merits of the Concentric and Spiral Approach

Application of this approach will make social science a subject of immediate and real interest for the average pupil. It will be the basis of correlating phenomena and happening with the immediate life of humanity, and for those who are more intellectual, it will be the basis of which academic insights and specialization in the discipline concerned can be attempted.

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Demerits of the Concentric and Spiral Approach

1. If proper care is not taken by the curriculum framers in gradually expending the material without mere repetition and if teachers do not carefully increase the scope of study in psychologically sound and natural way, this approach may cause monotony and lack of interest because of the repetitions of information.
2. At the initial stage, it will be difficult to give a clear picture of problem by presenting all the relevant details and by considering it in its totality.
3. It is difficult to develop a sense in the pupils.

The Topical Approach

In this approach, selected topics of study suitable for the age, ability and interest of children are included in the curriculum, and each topic is dealt with completely in the class where introduction of each topics are linked together first by the teacher with the help of link lessons for the children of the age group 13+ (above 13). This approach is quite possible.

The curriculum maker takes particular topics as the central theme of social science learning at different levels of instructions. At each stage, the topics vary in accordance with the ability and interests of the children. In the primary classes, the child may start the study of the development of concrete and familiar things such as food, clothing, shelter and means of transportation. In the middle classes, he may be introduced to more important and more difficult topics like history of instructions and of government. In the secondary classes, the student may be provided with ideas about ideologies like communism, socialism, capitalism, etc.

In this approach, we can deal with all the aspects of a problem and give an overall view of that particular problem.

Merits of the Topical Approach

1. This approach provides an action plan for dealing with vast material in a logical and rational way. It helps the pupils to understand the facts of their developmental setting.
2. This approach can be adapted according to the age, ability and aptitude of the children.
3. It imparts a sense of purpose to the pupils because of the total perception attempted.
4. This approach enables the teacher to control the subject-matter and adapt it to the varying needs of the children.

Demerits of the Topical Approach

1. It destroys the continuity of subject-matter.
2. 2. Since many aspects involved in a topic may be beyond the cognitive competencies of pupils in lower classes, a complete study of the topic will not be possible because of the above reason.
3. Generally speaking, when a topic is complex, very large and involve units posing varied levels of difficulty, it will be advisable to have the unit approach. The only thing is that care should be taken to effectively link all the units of the same topics as and when opportunities arise
4. There are several approaches to the curriculum development. They vary in their major focus. Concentric and spiral curriculum is concerned with the mental development of pupil. After topics have been selected according to the relevant fundamental principles described above,

they have to be systematically arranged so as to facilitate meaningful and effective transaction. The content should be arranged in a systematic manner.

Practice Questions

1. Differentiate between Topical and Concentric approach of curriculum development.

2. "For continuity of subject-matter, concentric approach of curriculum development is best." Justify.

Pedagogical Analysis and Unit Planning

Introduction

Teachers' knowledge of the pedagogy is decisive factor to students' achievement. Here, teacher's knowledge compromises of both content knowledge or pedagogical content knowledge. Acquiring only Mathematics content knowledge cannot prepare prospective Mathematics teachers for teaching mathematics. On the same lines, by just acquiring pedagogical knowledge alone, one cannot prepare prospective Mathematics teachers for teaching mathematics. This chapter discusses the concept, process and need of pedagogical analyses in mathematics teaching.

Concept of Pedagogical Analysis

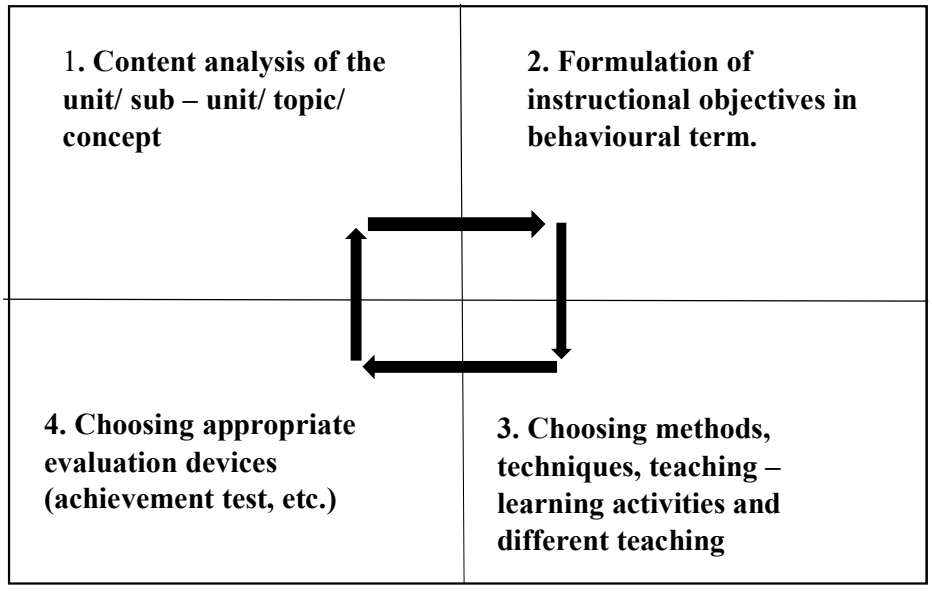
Pedagogical analysis (a composition of two words 'Pedagogy' and 'Analysis') stands for a type of analysis based on pedagogy. The dictionary defines "pedagogy" as the science of teaching. The phrase "Science of teaching" for the ways and means provided to or utilized by a teacher for managing his/her task of teaching as smoothly and effectively as possible by involving his/her least efforts for drawing maximum possible better teaching outcomes. Pedagogical analysis, based on four essential pillars along with their inherent mutual relationship and interdependence, are being considered essential in the effective teaching-learning process.

Components of Pedagogical Analysis

To have pedagogical analysis of the content, a teacher has to do unit analysis first; the instructional objectives are framed. After that learning experience, difference methods and techniques will be sorted out. Lastly, teacher thinks over the assessment technique for evaluation of subject-matter that has to be taught.

Concluding statements will be that the science of teaching, pedagogy is based on four pillars of teaching-learning process.

Four – fold Activities of Pedagogical Analysis



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Content analysis:

Content analysis is a meaningful division of content into units, sub-units and single concepts. It means, here, teacher divides the major theme into minor theme, sub-topics and simple concept. Thus, we can define content analysis as breaking down of the content or subject-matter with a sole objective of its proper, systematic, ordered and meaningful organization.

Formulation of objectives:

After going through the teaching content related to the topic, a teacher is expected to formulate the instructional objectives in the behavioral term as students are expected to demonstrate the specific type of behavioural outcome. So, formulation of instructional objectives in behavioural terms is the second step of the pedagogical analysis.

Learning experience and chosen methods:

It is the third step of pedagogical analysis, in which teacher provides teaching-learning experience with the help of chosen methods. The learning experiences tell how these formulated instructional objectives can be achieved properly. A teacher can go on smoothly in his teaching task with the help of selected methods, devices, techniques and aid materials. Here, science of teaching helps the teacher in thinking about best possible methods, strategies, techniques to be employed aid materials and likewise resources to be utilized for the teaching of the topic in the existing teaching-learning experiences.

Evaluation devices:

This is the last and foremost step of pedagogical analysis. Evaluation determines how properly teacher is proceeding in his/her task of teaching. To find out whether the set teaching objectives are achieved or not, we need testing devices known as evaluation. Thus, evaluation is the desired change in the behaviour of the students. It is the sum total of student's personality. The total behavioural outcomes are measured with help of evaluation devices.

To conclude, when a Mathematics teacher is asked to perform pedagogical skill analysis of the content or a topic to be taught, he has to go through the above said four-fold activities of pedagogical analysis.

Unit Planning

A unit may have several chapters or lessons. Therefore, it may not be completed in one lecture of period. There are many topics in mathematics which cannot be completed in one class. It may take several periods to complete the unit of knowledge.

Very often, a new teacher do not have enough practice to evaluate a unit in regard to how much subject-matter suits the varying age level of the students.

A unit plan enables a teacher to plan a unit of knowledge into small lessons to be covered in speculated period of time. It helps the teacher to organize the content matter of unit in understandable arrangement. Such unit plans are also known as teaching units.

Steps of Unit Plan

Unit 3 (C): Pedagogical Analysis, Unit Planning and Lesson Planning.

A unit plan may be written in the followings steps:

- **Subject:** Arithmetic, Algebra, Geometry, Trigonometry, etc.
- **Topic**: Heading of the Unit
- **Class:** To whom the unit is to be taught
- **Time:** Number of class periods
- **Aids:** To be used during entire unit
- **Content:** To be covered

i. Class: _____ Topic: _____

ii. Teacher's Activity

iii. Student's Activity

iv. Assignments

v. Evaluation

- **Objectives:** To be achieved after completing the unit.

Sample Unit Plan

Subject: Modern Mathematics **Class**: VIII

Topic: Set Theory **Time**: 4 periods of 40 minutes each

Aids: Charts showing tea set, sofa set, written alphabets, stamp fixed on a chart, etc.

Sub-units:

1. Meaning, definition of set and ways of describing set
2. Types of sets
3. Set operations
4. Problems based on sets

Objectives of the Unit:

1. To enable the students to understand the meaning of set.
2. To help the students to understand how set notations are used in every day life.
3. To enable the students to know different types of sets.

Sub-units:

A full-fledged lesson plan will be prepared on all the sub-units and will be delivered in speculated time.

Evaluation:

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Class test or unit test will be conducted to check the achievement of objectives of the unit.

Practice Questions

1. What is pedagogical analysis?
2. Elaborate the significance of pedagogical analysis in teaching-learning process.
3. Explain the concept of unit plan and its objectives.

Methods and Techniques of Mathematics Teaching

Introduction

The teaching and learning of mathematics have always been a major concern in education. Various commissions and committees have laid great emphasis on raising the quality of instruction in mathematics.

The National Policy of Education (1986) lays down the importance of mathematics as a vehicle for developing creativity. Recent researches in the area of learning have led to a deeper understanding of "how pupils learn". As a result, a broad range of new approaches to the teaching of mathematics have been suggested to achieve optimal learning. The highly structured nature of mathematics, its language and methods of proof have also attracted the attention of psychologists and educationists.

Consequently, the old methods of mathematics teaching which relied heavily upon rote learning and drill have been replaced by methods which rely upon discovery and problem-solving approaches.

The present unit presents the various approaches and techniques of teaching mathematics which will help the teacher to plan instruction in the classroom in a most effective manner.

Structure of Teaching Process

Effective teaching is not just a set generic practice of standing and talking about a topic while using black board. But, instead it is a set of context-driven intelligent decisions about teaching process to be adopted for teaching a particular topic. This decision-making process undertaken by teacher is blend of art and science of teaching.

Effective teachers do not use same set of methods, techniques, models, approaches and strategies in every lesson. They constantly reflect on their work, observe the learning by students and then adjust their practices accordingly.

In general, the structure of teaching process has a dynamic interrelationship between teaching objectives, teaching models, teaching methods, teaching techniques, teaching strategies and teaching approaches.

As the readers of this book are students of education, they must understand these concepts independently and in relation to each other. So, before discussing the specific methods for teaching mathematics, we must have insight into essential elements of teaching.

Elements of Teaching Process

Effective teaching is decision-making about choosing one or combinations of the following elements of a teaching process.

Teaching Approach:

Approach is equal to assumption. An Approach is a set of correlative assumption about the nature of content of the subject and its learning. It is treating something in a certain way. Teaching approach is your own personal philosophy of teaching. Approach is a set of

assumption (why). It is something that reflects a certain model or research paradigm. It is an understanding of theories and principles. It is the way teacher views the learning process, in which there are learning strategies with all his theories. Learning approaches that can be divided into two approaches are student-centered approach and teacher-centered approach.

Strategy:

Strategy usually requires some sort of planning for setting goals. Learning strategies of a teacher are learning activities undertaken with the aim of the learning process that takes place in the classroom and can achieve (goals), effectively and efficiently. In principle, the learning strategy of conceptual plans that will be the decisions to be taken in the learning process. Viewed from the side of the strategy, it can be grouped into two general categories: exposition-discovery learning and group-individual learning. Learning strategy is still conceptual, necessary for the implementation of certain teaching methods.

Method:

Method can be considered as a way of learning that must be taken to realize the plan that has been in thoughts of teacher in form of real and practical activities in the classroom to achieve learning goals. Thus, the strategy is "a plan for achieving goals" while the method is "a way for achieving goals". Method refers to a settled kind of procedure, usually according to a definite, established, logical, or systematic plan. It is a general way in which activity is conducted. A method is a plan for presenting the mathematics material to be learned and should be based upon a selected approach Refers to how you apply your answers from the questions to your day-to-day instruction in front of your students. Method is defined as a habitual, logical or prescribed practice or systematic process of achieving certain and results which accuracy and efficiency, usually in a preordained sequence of steps. A method is how to carry out these assumption and theories (how). It is a set of procedures that describe how to teach a subject. It is the way you apply these theories and principles. A method is an overall plan for the orderly presentation of language material, no part of which contradict, and all of which is based upon the selected approach. There are many methods of learning: lecture, inductive-deductive, analyses-synthesis, demonstration, discussion, simulation, laboratory, field experience, brainstorming, debates, symposium, and so forth.

Technique:

Technique is the various methods and processes developed through knowledge, skill and experience. It is a very specific, concrete stratagem or trick designed to accomplish an immediate objective. It is a procedure or skill for completing a specific task. Teaching techniques are little sneaky tricks we all know and use to get the job done in the classroom Technique means a systematic procedure, formula, or routine by which a task is accomplished. Techniques are steps to achieve certain goals Technique is a classroom device or activity and it is more specific than method. A technique is the tools and tasks you use to make your method succeed. A technique is implementation Techniques must be consistent with a method and therefore in harmony with an approach. Technique is a practical method or art applied to some particular task or skillfulness in the command of fundamentals deriving from practice and familiarity. Learning techniques is the way in which a teacher is carrying out the method of learning.

Model:

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Unit 4: Methods and Techniques of Mathematics Teaching

Learning model is a frame from the application of an approach, strategy, method and technique of learning. In learning, model is series of strategies, methods and techniques of learning in a single unified whole. Thus, the learning model is basically a form of learning which is reflected from start to finish which is typically presented by the teacher.

For effective teaching, a teacher must make reflective decisions about the alternatives he can make use to accomplish student learning. All the models, strategies, methods, techniques, or skills do not work equally well with all types of learners or all types of course content. The teacher must be aware about the options available and make intelligent judgements so as to bring about desired outcomes. Different aspects to be considered while selecting the teaching models, Strategies and methods are as follows:

- Learning outcomes and experiences desired
- Sequence of learning
- Learner's characteristics
- Setting and maintaining the climate of the classroom
- Making the purpose clear to the learners
- Organizing and making use of learning resources
- Open to expression of ideas
- Recognizing and accepting own limitations at personal level as well as practical level.

Practice Questions

1. Explain the structure of teaching process.
2. What are the elements of teaching process?
3. Relate the concept of approaches, methods, techniques and models in teaching process.
4. What are the different aspects to be considered while selecting teaching model, strategy, method and approach?

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Learner – Centered Methods

Introduction

Learner-centered teaching is an approach to mathematics instruction that places heavy emphasis on the students taking responsibility for problem solving and inquiry. The teacher is viewed as a facilitator by posing problems and guiding students as they work with partners toward creating a solution.

Inductive Deductive Method (Teaching Generalizations)

Inductive-deductive method is a combination of two separate method inductive method and deductive method. To know about these combinations of methods, let us study them separately one by one.

Inductive Method

This method is based on induction which means reaching to law or to generalization by showing that if it is true of any particular case and is further true for a reasonably adequate number of cases, then it is true for all such cases. In this method, we proceed from particular to general from concrete cases to abstract rules, and from the special example to general formula. The results are always generalized by studying particular, concrete cases and examples. Here, conclusions are based on the repetition of a particular kind of experience for so many times.

Using Induction in Mathematics

Examples of induction-based teaching of certain topics:

1. Property of sum of angles of a triangle For this, the teacher may ask the students to construct different type of triangles or he may provide them some cut-outs of different types of triangle. Let the students measure the angles of triangle, one by one and find sum of the angles in each case. And after the students have done measuring and summing, ask them to observe the result in all their angles in given the cases. This may lead to conclude that the sum of angles of a triangle is equal to two right angles or 180°.

2. Suppose the teacher wants to teach simple interest to students. Give them following two examples.

Find the simple interest on 100 for one year at 4% per annum.

S.I. = $\frac{100 \times 1 \times 4}{100}$ =₹4

Find the simple interest on ₹200 at 5% per annum for 3 years.

S.I. = $\frac{200 \times 5 \times 3}{100}$ = 30

Therefore, students may conclude that Simple Interest = $\frac{\text{Principle x Rate} \times \text{Time}}{100}$

3. Suppose the teacher wants to give the knowledge about the vertical opposite angles. The students may be asked to draw two intersecting lines and measure the vertically opposite angle, which will be equal in all cases when the lines are inclined at different angles.

4. Student may be given number of cases, e.g., 72-52, 112-92 and ask them to find the value in each case. Then they may be asked to find the values of (75)(7+5)(11-9) (11+9), (a + b)(a – b) by multiplication. Then they may be helped in generalizing that (Ist term)² - (II term)2 = (Ist term + IInd term) (Ist term - IInd term).

Merits of Inductive Method

1. In this method as, mathematical principle is established through a number of examples. Therefore, this method leaves no doubt in the mind of the students.
2. It encourages discovery reasoning and thinking power.
3. As students take interest in this method, it is based on the psychology of the child and principle of learning. The maxim "learning by doing" forms the basis of this method.
4. It is a scientific method as students arrive at a definite conclusion after systematic enquiry and investigation.
5. It discourages memorization. Child knows the procedure of discovering the formula. Therefore, he does not depend on his memory power.
6. Student participation in the teaching-learning process is maximum.
7. It helps in developing the intellectual power of the students and they become self-dependent.

8. It encourages self-study on the part of the students.
9. At the time of introducing the lesson, this method is very useful.
10. At the lower classes, this method is most appropriate as students can learn through their own direct experiences.
11. In this method, pupil-teacher relationship becomes intimate which helps in smoothening teaching-learning process of mathematics.
12. Problem of indiscipline is solved, as students are always busy in doing some practical work.
13. It encourages self-activity on the part of the students.
14. It is stimulating and motivating method.
15. The 'how' and 'why' of the rule of generalization are made clear through reasoning.
16. It starts from observation and direct experiences, and ends with developing a rule in the abstract form.
17. Homework becomes less in this method.

Demerits of Inductive Method

1. Inductive reasoning is not absolutely conclusive. It only establishes a certain degree of probability, which increases with the number of facts observed. Hence, it cannot be used for exact mathematical demonstration of course. It can be employed for finding mathematical facts.
2. At the higher classes, this method is not useful as the unnecessary details may make the teaching dull.
3. No doubt, this method helps in discovering the formula with the help of large number of examples, but to fix the knowledge of the topic in the mind of the student, a proper practice is also needed. This practice cannot be given with this method.
4. 4. Other reason for the non-practicability of this method at the higher class is that at higher classes abstract concepts are to be developed instead of concrete concepts.
5. It is time-consuming and uneconomical method, as it demands too much time from the students and teachers.

6. This method demands the preparation of some charts, models or other teaching aids from the teacher. But mostly, teachers stick to the method they themselves were taught by and use only the techniques (chalk and talk) they feel most at home with.

Deductive Method

Deductive method is just the reverse of inductive method. In this method, we proceed from general to particular, abstract to concrete and formula, and then they are asked to apply this formula in the solution of some particular and related problems in mathematics. In other words, child does not discover laws but develops the skills in applying them. He is provided with information of facts, principles and the theories.

Examples of deduction-based teaching of certain topics:

1. Property of sum of angles of triangle: While teaching this topic by using deduction, teacher may simply introduce the angle sum property of a triangle and ask students to apply the property in finding solution to problems. Then he will give few problems to students related with this proposition.
2. Simple interest: Suppose the teacher is going to teach simple interest to the students. Then he will say, well students, today I will teach you simple interest and then he will write the following formula on the blackboard.

$$\text{Simple Interest} = \frac{\text{Principle x Rate x Time}}{100}$$

By explaining the application of this formula, the teacher will now solve some problems. Then he will give few problems to students and will ask to solve these problems with the help of this formula.

Merits of Deductive Method

1. As solution of problems with the help of given formula does not take much time, this method is economical.
2. As it starts with rule and provides for practice, this method is very much useful at the practice and revision stage.
3. At the higher classes, this method is very suitable method.
4. It helps in increasing the speed and efficiency in solving the numerical problems.

Demerits of Deductive Method

1. It is very difficult for a beginner to understand an abstract piece of mathematical work, if not proceeded by a number of concrete examples.
2. It is not a psychological method. Students are compelled to learn so many facts without seeing their capacity to digest.
3. As the steps are not clear, it creates so many doubts in the mind of the students.
4. In this method, students are supposed to memorize so many formulae and rules. If he forgets, he becomes helpless.
5. Every time, there is a strain on the mind of the students.
6. This method is not suitable for exploring the new field.
7. It does not help in developing the originality and creativity among the students as no discovery is possible by this method.
8. Due to its abstract nature, this method is not very much helpful for the lower classes.
9. There is no student's participation in this method. They remain passive throughout the period.
10. It is illogical and mechanical method, and does not help in the development of independent thinking of the students.
11. Pupil teacher relationship does not become intimate in this method.

Conclusion

In conclusion, we can say that the methods have their own merits and demerits. Both the methods are partner of each other, and if combined, they remove the incompleteness and inadequate of each other. Therefore, at the time of teaching to get maximum advantage of both, a teacher should try to combine both the methods. As in the process of learning of a topic, there are two parts - establishment of a formula and application of that formula, the teacher should apply inductive method at the time of establishing the formula. Thus, teaching of the teacher must begin with inductive method and it must end with the deductive method of teaching mathematics.

Analytic-Synthetic Method (Teaching Proofs)

Analytic and Synthetic method is a combination of two methods- analytic method and synthetic method. To understand this combination, we have to study these two methods separately.

Analytic Method

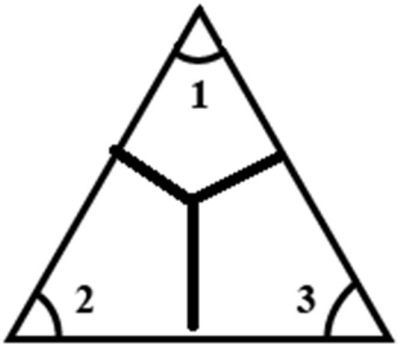
Analytic method has been derived from the word “analysis” and word “analysis” is a process of breaking a thing into its smaller parts. In this method, we break the problem into simpler parts such that its solution may become simple. In analytic method, we move from unknown part of the problem by adopting the process of analysis. We start with what is to be proved and then by going step by step analytically, we connect the unknown part with the known part. In other words, we proceed from conclusion to hypothesis. In fact, the students work backward in this method.

Using Analytical Method in Teaching Mathematics

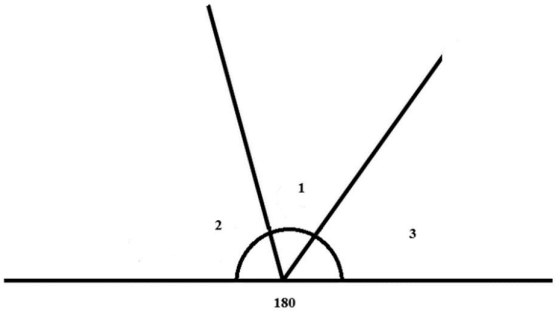
Examples of analyses based teaching of certain topics:

1. Angle sum property of triangle:

Ask pupils to draw a number of triangles. Ask them to measure the angles of each triangle and find their sum.



You can also ask children to cut the three corners of the triangles and put them at a point so that they form a straight line.



Conclusion: The sum of 3 angles of a triangle is 180°.

2. Suppose teacher wants to teach how to prove the following identity:

Question: If $= \frac{x}{y} = \frac{p}{a}$, then prove that $= \frac{xp-7y^2}{y} = \frac{p^2-7yq}{q}$

Proof: For using analyses method, we need to begin from unknown part, i.e., to prove part.

$$\frac{xp-7y^2}{y} = \frac{p^2-7yq}{q}$$

Cross multiplying, we get

xpq - $7y^2$q = yp^2 - $7y^2$q

Cancelling - $7y^2$q on both sides, we get

xpq = yp^2

Cancelling p from both sides, we get

xq = yp

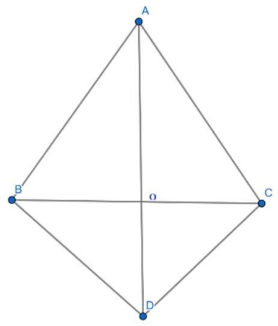
Rearranging, we get

$\frac{x}{y} = \frac{p}{a}$

Hence, by going back step-by-step from unknown part, we can prove the identity.

3. Suppose the teacher wants to prove the following identity:

If △ABC and △BCD are two isosceles triangles on the opposite sides of the same base, prove that the line joining their vertices bisect the base BC at right angle.



Teacher: Look at the figure, and read the statement. What we have to prove?

Student: We have to prove that AD bisects BC at right angle (unknown).

Teacher: How we can prove this?

Student: We can prove this by proving ∠ BOA = 900.

Teacher: We shall prove it by means of two congruent triangles.

In other words, we must prove ΔBOA ≅ ΔCOA

As you know, we cannot prove the equality of two triangles directly. Therefore, first, we shall prove ΔABD ≅ ΔACD.

In Δ ABD and Δ ACD, AB = AC, BD = CD and AD = AD (isosceles triangle).

So, by Side-side-side property, Δ ABD ≅ Δ ACD, ∠ BAO = ∠ CAO (C.P.C.T. = Corresponding Parts of Congruent Triangle).

Now, we can prove Δ BOA ≅ Δ COA, by side-angle-side property.

As ∠BAO = ∠CAO, AB = AC and AO = AO

In Δ BOA and Δ COA, ∠BOA = ∠ COA, BO = CO (C.P.C.T.)

∠BOA + ∠COA= 180° (Linear Pair)

2∠BOA = 180° (as ∠BOA = ∠COA)

So, ∠BOA = 90°

Therefore, BOA = 90° = ∠ COA

Hence, AD bisects the base BC at right angles

Merits of Analytic Method

1. This method helps in understanding and leaves no doubts in the mind of the students.

2. This method is psychological as students take interest in it.
3. This is a scientific method as it leads to spirit of enquiry and investigation.
4. It develops reasoning and thinking power.
5. It develops originality and creativity among the students.
6. This method helps the students to become self-confident and self-reliant as they discover the knowledge with their own efforts.
7. It discourages memorization on the part of the students.
8. There is a regular sequence in this method as each step has its own reason and purpose.
9. Student participation is maximum in this method.
10. It helps in developing heuristic attitude among the students as it is a method of discovery and demands thought.
11. In this method, we apply inductive reasoning.
12. As students remain busy in work, there is no problem of indiscipline.
13. It builds up scientific attitude, originality and creativity among students.
14. The teacher acts as a guide and plans solutions for discovering learning by students.
15. Homework becomes less.

Demerits of Analytic Method

1. This method is lengthy, laborious and time-consuming method.
2. Students cannot apply this method to all topics.
3. Books are not written on analytic lines.
4. At the practice stage, this method is not very useful, as this method is not very useful, as this method is forerunner of synthetic methods. In other words, this method is not very useful only in the beginning for discovering the solution of the problems.

Synthetic Method

This method is opposite of the analytic method. In this method, we proceed from known part to unknown part of the problem. Synthesis means to combine

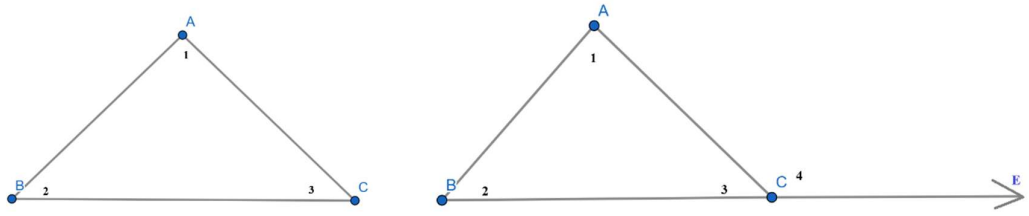
separate elements to get something new. It proceeds from data to what one wants to prove or from hypothesis to conclusion. The students work forward in this method. In this method, what is already given or known is arranged in such a way that the synthesized picture may lead us to reach the point where unknown information becomes true and gives the desired result or conclusion.

Using Synthetic Method in Mathematics Teaching

Examples of synthetic-based teaching of certain topics:

1. Prove that sum of angles of triangle is equal to two right angles:

Here, the knowledge of results related to angles such as alternate angles, corresponding angles, angle pairs, etc. already proved prior to proving the given proposition.



Step 1: Let ABC be the triangle with angles 1, 2 and 3.

To prove: ∠ 1+ ∠ 2+ ∠ 3 = 2 right angles = 180°.

We know that a linear pair measures 180°.

For having a linear pair in Δ ABC, we extend base BC to E.

Now, ∠ 3 and ∠ 4 form a linear pair.

So, to prove, ∠ 1+ ∠ 2+ ∠ 3= ∠ 3+ ∠ 4

i.e., ∠ 1+ ∠ 2 = ∠ 4

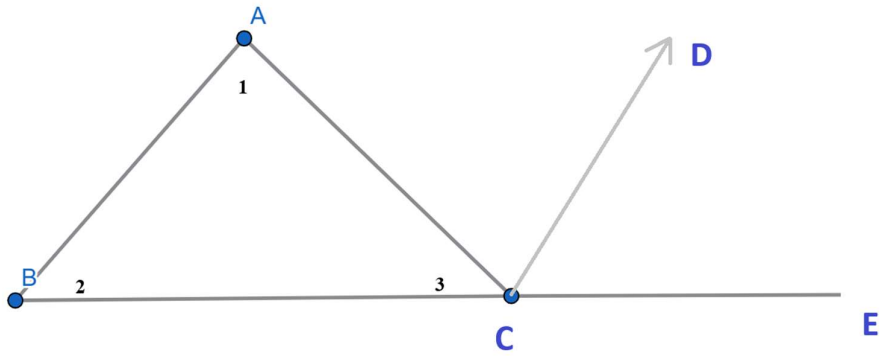
We can at least cut an angle equal to ∠ 2 from ∠ 4

if we draw a line CD parallel to BA (corresponding angles are equal).

∠2 = ∠DCE

Construction: Draw CD ||BA.

Proof



AB || CD (const.) and BE meets them.

∠ABC = ∠DCE (corresponding angles)..(i)

Again, AB || CD and AC meets them

∠BAC = ∠ACD (alternate angles)..(ii)

Adding (i) and (ii),

∠ABC+∠BAC = ∠DCE + ∠ACD

Adding ABC to both sides,

∠ABC + ∠BAC + ∠ BCA = ∠DCE + ∠ACD + ∠ACB = 2 right angles= 180°

2. Suppose the teacher wants to teach how to prove the following identity:

If $\frac{x}{y} = \frac{p}{a}$, then prove that $= \frac{xp-7y^2}{y} = \frac{p^2-7\,yq}{q}$

Now, as synthesis starts from known part

$\frac{x}{y} = \frac{p}{q}$ (given)

By subtracting $\frac{7y}{p}$ from both sides (why it is not explained).

$\frac{x}{y} - \frac{7y}{p} = \frac{p}{q} - \frac{7y}{p}$

$$\frac{xp - 7y^2}{y} = \frac{p^2 - 7\,yq}{qp}$$

Cancelling from the denominator from both sides,

$$\frac{xp - 7y^2}{y} = \frac{p^2 - 7\,yq}{q}$$

Hence proved.

Merits of Synthetic Method

1. Student can acquire skill, speed and efficiency with this method.
2. This method is short, concise and time-saving method.
3. At the practice stage, this method is very useful at this method is a follower of analytic method.
4. It is a method in which we present the facts which have already been discovered.

Therefore, this method is a product of thought.

Demerits of Synthetic Method

1. This method leaves so many doubts in the mind of the students and gives no explanation for them.
2. There is no clear understanding of the topic.
3. Although each step is correct but for its sequence, we have no reason, i.e., why this all happening is not explained in this method.
4. The student becomes nervous when some new problem is given.
5. This method does not develop the reasoning and thinking power of the students.
6. This method does not help the students to become self-confident and self-reliant as there is no scope for independent discovery.
7. This method does not develop the originality and creativity.
8. It is an unscientific method, as it does not help in developing heuristic attitude among the students.
9. This method encourages memorization. Therefore, if the students forget one step, it is not easy for him to proceed further.
10. There is no active participation of the students in this method.
11. As the teacher is not in touch with the students, there may be a problem of indiscipline while teaching with this method.

In conclusion, we can say that both the methods are complementary to each other. Analysis helps in understanding and synthesis helps in understanding the new material, but for fixing and retaining what has been understood, the synthetic method should be used. Although the two are independent, but both should be used in combination to get good result.

Practice Questions

1. Discuss analytic method of teaching mathematics with respect to its procedure, merits, demerits and practical applications. What practical difficulties the teacher may confront in its application?
2. Discuss inductive method of teaching mathematics with special reference to its meaning, procedure, merits, demerits and practical applications.

Activity – Centered Methods

Introduction

The idea of activity-centered methods is rooted in the notion that children are active learners rather that passive recipients of information. The aim of activity-centered methods is for learners to construct process of self-learning and problem solving, and transfer of information and skills.

Problem-solving Method

The problem-solving method or briefly problem methods consists in training the pupils to solve problems The teacher presents a problem, which challenges the intellect of the students. They feel like solving the problem. Now, the problem is a felt-difficulty in the attainment of an objective. In the words of Yoakam and Simpson, "a problem occurs in a situation in which a felt-difficulty to act is realized. It is difficulty that is clearly present and recognized by the thinker. It may be a purely mental difficulty or it may be physical, and involves the manipulation of data. The distinguish thing about a problem however, is that it impresses the individual who meets it for finding a solution." As mathematics is a subject of problems, training in problem solving is most essential.

Procedure of Problem-solving Method

1. Recognizing the problem: First of all, we sense the presence of a problem and recognize the type of the problem.
2. Defining the problem: Defining the problem refers to putting boundary around the problem so that it does not overlap with similar problem. In other words, writing the problem in precise words is called defining the problem.
3. Collecting relevant data: Next, all sort of relevant data has to be recalled which can be helpful in solving the problem.
4. Organizing the data: Then the data is to be so organized that it can lead to the solution of the problem.
5. Formulating the tentative solution: Out of the tentative solution, a correct solution is to ✓ be found out by a process of reasoning.
6. Verifying the result: In Mathematics, we do not accept a conclusion without proper verification. The students are required to verify the conclusion by reversing the process of reasoning.

Example

Define union of two sets. If A = {2, 3, 5} B = {3, 5, 6} and C = {4, 6, 8, 9} then prove that A ∪ (B ∪ C)=(A ∪ B) ∪ C .

Solution:

Step 1: Identifying and Defining the Problem

After selecting and understanding the problem, the child will be able to define the problem in his own words that:

I) The union of two sets A and B is the set, which contains all the members of a set A and all the members of a set B.

II) The union of two sets A and B is expressed as 'A ∪ B' and symbolically represented as A ∪ B = {x ; x ∈ A or x ∈ B}.

III) The common elements are taken only once in the union of two sets.

Step 2: Analyzing the Problem

After defining the problem in his own words, the child will analyze the given problem that how the problem can be solved.

Step 3: Formulating Tentative Hypothesis

After analyzing the various aspects of the problem, he will be able to make hypothesis that first of all, he should calculate the union of sets B and C, i.e., (B ∪ C) and then the union of set A and B ∪ C.

Thus, he can get the value of A ∪ (B ∪ C). Similarly, he can solve (A ∪ B) U ∪ C.

Step 4: Testing Hypothesis

Thus, on the basis of given data, the child will be able to solve the problem in the following manner.

In the example, it is given that:

B ∪ C = {3,5,6} ∪ {4, 6, 8, 9}

= {3, 4, 5, 6, 8, 9}

A∪ (B ∪ C) = {2, 3, 5} ∪ {3, 4, 5, 6, 8, 9}

= {2, 3, 4, 5, 6, 8, 9}

Similarly,

$A \cup B = \{2, 3, 5, 6\}$

$(A \cup B) \cup C = \{2, 3, 4, 5, 6, 8, 9\}$

After solving the problem, the child will analyze the result on the basis of given data and verify his hypothesis whether $A \cup (B \cup C)$ is equal to $(A \cup B) \cup C$ or not.

Step 5: Verifying of the Result

After testing and verifying this hypothesis, the child will be able to conclude that $A \cup (B \cup C) = (A \cup B) \cup C$.

Example

Define union of two sets. If $A = \{2, 3, 5\}$ $B = \{3, 5, 6\}$ and $C = \{4, 6, 8, 9\}$ then prove that $A \cup (B \cup C) = (A \cup B) \cup C$.

Thus, the child generalizes the results and applies his knowledge in new situations.

For example, the problem is to find the area of the four walls of the room. While analyzing the problem, it gets connected with the previous knowledge that area of the four walls can be connected with the area of a rectangle. The area of the four walls is the total area of six rectangles forming the walls. Moreover, area of the opposite walls is equal. This reduces to finding the area of three different faces and doublings. This leads to the framing of the formula. That is to be verified by applying the formula to solve new problem.

Merits of Problem Solving Method

1. It prepares the pupils in problem solving. The training in solving mathematical problem is related to solving problems of life.
2. It stimulates thinking, reasoning and imagination of the students.
3. It develops habit of independent work and initiative in the students.
4. This method suits teaching of mathematics as it is full of problems.
5. It stimulates intellectual curiosity and motivates the students to exert further.
6. It provides scope for individual work and habit of self-study.
7. The student learns by doing from his own exercise. Such learning is well retained in the mind

8. There is a close pupil-teacher contact. The teacher understands the doubts and difficulties of individual pupils, and provides them guidance individually.
9. It develops desirable qualities like patience, cooperation, self-confidence, etc.

Demerits of Problem Solving Method

1. This method is slow. The students progress slowly.
2. It is not suitable for all topics of mathematics.
3. It is not suitable for students of lower classes as they are not sufficiently mature for the purpose.
4. This method requires special preparation on the part of the teacher. An average teacher may find it difficult to use this method.
5. It is also difficult to organize the content of mathematics according to the requirements of this method.
6. Suitable textbooks for this method are not available.

Lecture-cum-Demonstration Method

Lecture-cum-demonstration includes the merits of the lecture as well as demonstration method. It attempts to filter out the disadvantages of both. Demonstration means 'to show'. In Lecture method, teacher just tells, but in demonstration method, teacher shows and illustrates certain fundamental phenomena.

Lecture Method:

Lecture method is most convenient and inexpensive method of teaching concepts any subject. It hardly requires the use of scientific apparatus, experiment and aids materials except the blackboard. It is teacher-controlled and information-centered approach in which teacher works as a role resource in classroom instruction. In this method, the only teacher does the talking and the student is passive listener. This creates dullness in the classrooms as the interaction between the pupil and teacher ceases 91 to occur.

In the field of education, lecture method is used very frequently. This method is used in order to acquire knowledge and concept. It mainly focuses on cognitive objectives. The main emphasis of this strategy is the presentation of the content. In this method, teachers plans and controls the whole teaching-learning process.

To make the lecture interesting, the teacher can take the help of audio-visual aids.

Demonstration Method:

The word demonstration means to give demos or to perform the particular activity or concept. In demonstration method, the teaching-learning process is carried in a systematic way. Demonstration often occurs when students have a hard time connecting theories to actual practice or when students are unable to understand applications of theories.

The dictionary meaning of the word "demonstration" is the outward showing of a feeling. etc.: a description and explanation by experiment; so also logically to prove the truth, or a practical display of a piece of equipment to show its capabilities. In short, it is a proof provided by logic, argument, etc. To define, it is a physical display of the form, outline or a substance of object or events for the purpose of increasing knowledge of such objects or events. Demonstration involves "showing what or showing how". Demonstration is relatively uncomplicated process in that it does not require extensive verbal elaboration.

Now, it will be easy to define what is lecture-cum-demonstration method. To begin with, this method includes the merits of lecture method and demonstration method. The teacher performs the experiment in the class and goes on explaining what she does it takes into account the active participation of the student and is thus not a lopsided process like the lecture method. The students see the actual apparatus and operations, and help the teacher in demonstrating experiments, thereby they feel interested in learning. This method also follows maxims from concrete to abstract wherein the students observe the demonstration critically and try to draw inferences. Thus, with help of lecture-cum-demonstration method, their power of observation and. reasoning are also exercised. So, the important principle on which this method works is "Truth is that works".

Requirements of Good Demonstration

For the success of any demonstration, the following points should be kept in mind:

(i) It should be planned and rehearsed by the teacher beforehand.

(ii) The apparatus used for demonstration should be big enough to be seen by the whole class If the class may be disciplined, she may allow them to sit on the benches to enable them better view.

(iii) Adequate lighting arrangements be made on demonstration table and a proper background table need to be provided.

(iv) All the pieces of apparatus be placed in order before starting the demonstration. The apparatus likely to be used should be placed on the left-hand side of the table and should be arranged in the same order in which it is likely to be used.

(v) Before actually starting the demonstration, a clear statement about the purpose demonstration be made to the students

(vi) The teacher makes sure that the lecture-cum-demonstration method leads to active participation of the students in the process of teaching.

(vii) The demonstration should be quick and slick, and should not appear to linger on unnecessarily.

(viii) The demonstration should be interesting so that it captures the attention of the students.

(ix) It would be better if the teacher demonstrates with materials or things the children handle in everyday life.

(x) For active participation of students, the teacher may call individual student in turn to help him in demonstration.

(xi) The teacher should write the summary of the principles arrived at because of demonstration on the blackboard. The blackboard can be also used for drawing the necessary diagrams.

Process of Lecture-cum-demonstration

1. Planning and presentation: While planning a demonstration, the following points should be kept in mind

- Subject-matter
- Lesson planning
- Rehearsal of experiment
- Collection and arrangement of apparatus

2. Introduction of lesson: The lesson may be introduced on the following basis:

- Student's personal experience
- Student's environment

- Telling story
- A simple and interesting experiment/demo

3. Presentation of the subject-matter:

- The teacher must study the subject-matter on broad basis taking into consideration the interest and experience of students.
- While demonstration is going on, questions should also be asked which help the students to understand the principles.
- The teacher should try to illustrate the facts and principles.
- Language used by teacher should be simple and clear.

4. Experimentation:

- Demonstration should be properly spaced, striking, clear and convincing.
- The demonstration table should have only apparatus.
- The experiment should be simple and speedy.
- All the apparatus should not be displayed at once.

5. Blackboard work:

A big blackboard behind the demonstration table is necessary in order to summarize the principles and other matters of demonstration, and also to necessary diagrams and sketches.

Merits of Lecture-cum-demonstration Method

- Demonstration method is economical as it helps in economizing resources.
- This method is psychological method as the students are shown concrete things.
- This method is especially useful where:

(i) The apparatus is expensive.

(ii) The experiment involves some danger.

(iii) The apparatus is sensitive to break.

- The experiment involves some difficult and complex operations.
- There is student participation.

- It saves time and effort.
- It is helpful to promote useful discussion.
- It is more efficient method.
- It is activity method.
- It is useful for all types of students.
- It is helpful for teacher.

Demerits of Lecture-cum-demonstration Method

- **Ignore maxim of education**: The maxim of education 'Learning by Doing' and the principles of psychology of learning has no place in this method.
- **Visibility**: Visibility is main problem for a teacher because all the students may not be able to see the details and results of a demonstration.
- **Speed of experiment**: Either too fast or too slow speed of demonstration sometimes may create trouble.
- **Ignore individual difference**: This method totally ignores the main principle of psychology.
- **Hinder progress**: This method somehow hinders the development of laboratory skills among the students.
- **Not useful for developing scientific attitude**: Not useful for developing scientific attitude: it does not provide opportunity to learner to use his own cognitive processes.

This method can prove to be one of the best methods for teaching to High and Higher Secondary classes. The teacher should encourage the students to demonstrate the experiments to the class.

Practice Questions

- Explain problem-solving method of teaching mathematics with the help of suitable examples. What practical difficulties the teacher may confront in its application?
- Explain lecture-cum-demonstration method in teaching of mathematics.

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Principal, TSCER

Techniques of Mathematics Teaching

INTRODUCTION

A technique implies the external mode or form in which teaching takes place from time to time. There is difference between the methods and techniques. A method is wider in scope whereas a technique is much narrow. Method stands only for a systematic arrangement of the material that is to be taught. But a technique is a contributor to the method.

There are certain important techniques such as oral work, written work, drill work, assignment, etc. which can be effectively used by mathematics teacher to make the teaching and learning process more meaningful and successful.

Drill and Review

Drill implies prior understanding of content and its appropriate application. It is one of the important devices of fixing an impression on the minds of the students. This is based on the principle of learning by doing and on the law of exercise. It is an admitted fact that practice makes a man perfect. Laws of learning show that learning is followed by forgetting. Therefore, in order that facts are retained in mind, these should be sufficient drill work.

Importance of Drill in Mathematics

1. Knowledge of mathematics becomes useful only when the students can apply that knowledge in the solution of the problem and this ability of applying the knowledge comes through drill work or practice.
2. Students get opportunity to work independently.
3. It helps in developing the self-confidence of the students, and in this way, they learn the art of solving even the most difficult problems of mathematics.
4. Foundation of learned material becomes solid through drill work.
5. The impression of the learned material through drill work becomes deep in the mind of the students.
6. It is helpful in retaining the knowledge as well as in learning of skill. In other words, it helps in revision and review work.
7. Drill work is very helpful in computation work and for learning geometrical figures.

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Principal, TSCER

8. It helps in achieving high speed and accuracy in solving mathematical problems.
9. Drill work also plays a great role in learning principles and formulae in arithmetic, algebra and theorems in geometry.
10. It is especially useful for below average and small children. With drill (practice), they begin to understand the learnt facts.

Suggestions for Effective Use

1. The material to be memorized or drill upon should be meaningful. Student must know why they are having drill in a particular topic.
2. Drill work should not be lengthy.
3. To make drill work interesting, there must be some variety in the drill work. In other words, it should be given in various from, e.g., oral work, written work, practical work, etc.
4. In drill work, the accuracy should always be given more importance than the speed of the work. Teacher must point out mistake, if any, and it must be corrected immediately.
5. It should not be given in the form of punishments. The student should be able to take pleasure in that.
6. While giving drill work, the principle of individual differences must be given due consideration as all the students do not possess equal ability and capacity to do drill work.
7. There must be some play activity, competition and group work in drill to make this work more interesting.
8. Drill work should be interesting, stimulating and sufficiently motivating. Otherwise, there will be no use of giving drill work.
9. In drill work, there must be proper evaluation of the achievement level of the student so that the teacher can make desired improvement in the drill work schedule.
10. In drill work, students should be encouraged to memorize the basic facts and operations of mathematics with clear understanding and there should not be any mechanical cramming.
11. While giving drill work, goals and objectives of teaching mathematics must be kept in mind.

12. Drill work should also be well planned. Teacher must learn when to give drill work and how much of it is needed at a particular stage.1
13. The drill work should always be based on the fact already taught. New facts or rules should not be given as drill work without prior teaching in class.
14. Drill work should be properly supervised. The students may commit some mistakes. Therefore, it should be then and there identified and corrected by the teacher.
15. Drill should follow learning and understanding of basics. It should not encourage role memorization without understanding the subject-matter.
16. Drill may also provide diagnostic information about pupils.

Assignment in Mathematics

- According to Good's Dictionary of Education (1973), homework means "School assignment to be completed out of regular school hours at the residence of the pupil. In other words, the daily work given by the teacher from the work done in the school in the form of solving some problems or to learn some principles, definitions and facts by heart or to write or draw some geometrical figures by the students at her home is called homework or home task.

Demerits of Home Assignment

1. Some educationists are of the view that homework should not be given to the students. They feel that if the school time is used sincerely and efficiently, homework does not remain necessary.
2. Generally, the teacher does not know how much work the other teacher has given. He only considers that child has to do only his homework in order to make his result. Thus, too much homework becomes extra burden for the students.
3. If the child does too much homework and remains busy all the time, it may even adversely affect the health of the child.
4. In quite a few child homes, the conditions may not be congenial for the child to do the homework.
5. Heavy homework snatches the leisure time pleasure of the young child, and in this way, we are committing some cruelty to children.
6. A large amount of homework may create a distaste for school in the child.

7. Some children develop emotional tensions because of homework.
8. Homework assignments are sometimes misused as punishments.
9. Fear of punishments for not doing homework gives birth to so many complications like the habit of copying, truancy, telling lie, etc. In addition to this, many children get the help of their elder brother, sister or parents to do their homework. It creates a dangerous result. Because if a student does some wrong thing in his notebook and teacher does not correct it, he grasps the same wrong thing. Only signing the notebook by teacher is not desirable.
10. Educationist like Bray emphasizes that after a long day's work at school, assigning the homework is not at all desirable. It will do more harm than good.

But this is one side of the picture. Homework has its importance also.

Importance of Home Assignment in Mathematics

1. Mathematics is such a subject, which can only be learnt by continuous application and practice. Therefore, in order that mathematical operations can be fixed in child's mind, homework is necessity.
2. School time is insufficient to exhaust everything in mathematics. Homework has to be given to supplement classroom teaching
3. It becomes all the more important and necessary in view of the heavy syllabus as compared to the time available.
4. It utilizes the leisure time of the children, which otherwise would have been wasted.
5. Some students do not have the habit of hard work. It may establish the habit of working hard and that too regular.
6. It supplements the classroom teaching, and is a practical means to cover the lengthy and heavy syllabus.
7. It may serve as a link for parent-teacher cooperation. It enables the parents to examine from time to time the work and progress of their children and give suggestion for further improvement.
8. It develops a sense of responsibility in the child.
9. It develops the habit of self-study in the child

10. Educationally also, it is sound principle that lesson read in the school must be revised at home.
11. It also develops good habits of regularity and punctuality.
12. Many educationists are of the view that five hours' work at school is insufficient and it must be supplemented by some work at home.

Therefore, from the above discussion, we can say very safely that it is not the homework which is objectionable, but it is the nature and amount of homework, and the way in which it is assigned, needs to be blamed. Some teachers assign too much homework, some do not take into consideration the capacity and abilities of the students, and some teachers can assign homework only to keep busy the students at home. Some teachers try to save their own labour and assign a heavy homework to cover the syllabus. But if we curb out the above tendencies of the teacher for assigning homework and homework is assigned with great care in a proper amount, then homework will sure bring some good results.

Suggestions for Effective Use

Dr. Lorence Fox in his book "Homework is What We Make It" has assigned following criteria:

1. It should be well graded. Its nature and amount must be determined by the individual's capacity and interest, and not by the will of the teacher.
2. It should be based on the teaching done in the class.
3. It should not be considered as a sort of punishment.
4. It should not be given by the teacher in a disturbed state of mind.
5. It should be challenging to the students.
6. Home conditions of the individual child must be kept in mind.
7. It should be related to every day life and interest of students.
8. It should encourage individual choice and creativity.
9. Home assignments s should be duly checked and corrected. Otherwise, students may develop habit of coping it.
10. If possible, remarks may be written and suggestions should be given about how to improve.

11. The teacher may make a list of common errors made by the students and discuss them in the class.
12. It should be well adjusted in available time.

In conclusion, it can be said that home assignments should be enjoyable, useful and attractive for the students. The principle of individual difference should be kept in mind, while assigning the task.

Practice Questions

- What is the place of drill work in teaching of mathematics? How will you make it effective?
- What is the place of homework in teaching of mathematics? What are the necessary precautions in assigning it to students? How will you make it more effective?

Learning Resources Mathematics Laboratory and Mathematics Club

Introduction

The constructivist learning approach calls for the extensive use of various learning resources as self-learning is emphasized in classrooms. Abundant use of learning resources gains attention of learners in the learning processes, and at the same time, helps the teacher to sustain the involvement of learners in learning. Apart from that, the learning turns into an enjoyable activity and all-round development of learners is assured both in cognitive and co-cognitive aspects. Keeping the relevance of learning resources in mind, the widespread use of learning resources is suggested both at elementary and secondary level. In Chapters 16, 17 and 18, we would learn about the use of learning resources for mathematics at school level.

Mathematics Laboratory

Learning resources also include mathematics laboratory (math lab) and mathematics corner, which slightly differ in their organization and material possessed. Let us discuss the major differences among them. "Mathematics laboratory is a unique room or place, with relevant and up-to-date equipment, known as instructional materials, designed for the teaching and learning of mathematics and other scientific or research work, whereby a trained and professionally qualified person (mathematics teacher) readily interacts with learners on specified set of instructions" (Adenegan, 2003). Math lab is a place where learners get opportunity to engage with mathematical objects, experiment mathematical theories, solve mathematical puzzles and problems, play mathematical games, experience hands-on training, and so on. The material or equipment that can be found in the mathematics laboratory includes, among others, constructed (wooden/metal/plastic made) mathematical sets, charts and pictures, computer(s), computer software, audio-visual instructional materials such as projector, electronic starboard, radio, television set, tape recorder, video tape, etc., solid shapes (real or model), bulletin board, three-dimensional aids, filmstrips, tape photographs, portable board or whiteboard, abacus, cardboards, tape measure, graphics, workbooks, graphs, flannel boards, flash cards, etc. (Adenegan, 2003).

Math lab consists of a number of materials and objects. Mathematics corner is a miniature form of math lab. Math lab is highly organized, consists of several objects/materials/instruments and requires specialized skills in developing them, but math corners are simple and contain few mathematical objects and items. You can set up a math corner at the corner of any other lab or at the corner of classrooms. Usually, math corner is a place where learners find the ordinary/common kinds of mathematical items and you can utilize these items during the classroom interaction. In a way, math corners include math-related teaching-learning aids. It is to be noted that the objects found in math labs can also be found in math corners.

Significance of Mathematics Laboratory

- "Learning by Doing" is one of the cardinal principles of teaching mathematics. It is, therefore, important that practical work should form a prominent feature in any mathematics course. For this, laboratory is essential in the school.

Dr. (Mrs.) Megha D. Gokhe
Principal, TSCER

- The things learnt by the students through purposeful activity are permanently affixed in the minds of the pupils. The knowledge imparted without experimental collaboration or evidence remains superficial, hazy and imperfect.
- It provides safe and proper place for placing all the essential materials and equipments concerning the learning activities in a subject.
- It provides a good platform for the integration of theory with practice in the subject of mathematics.
- It helps in creating interest of the student in the learning of mathematics.
- It helps in making use of all progressive methods like heuristic, inductive, analytic, laboratory, and teaching and learning of mathematics.

As theoretical concepts are easily clarified in laboratory, therefore, it saves the time and energy of the teachers and students.

- It helps in satisfying the creative and constructive urges of the students.
- It helps in satisfying the curiosity of the students.
- Acquiring knowledge and heuristic attitude, which are two main objectives of teaching mathematics, can be achieved through practical work as Laboratory.
- Laboratory work helps in training the students for the practical application of mathematical facts and principles in their life.
- It provides opportunities for making geometrical diagrams, charts and models, and accustoms the students to use various tools, e.g., calculators, computers, etc.
- While doing practical work, the students follow many books, habits like cooperation, resourcefulness, initiative, self-confidence, self-reliance, etc. which are applicable in the daily life of the students.
- It is helpful in meeting the needs and interest of gifted and brilliant students in the subject of mathematics.
- Whenever a student handles or works with concrete objects, his learning is quick and understanding is better. For example, handling of three-dimensional objects is a valuable experience, which assists in cultivating space perception and a more accurate formulation of space concepts.
- Construction and demonstration of working models in geometry helps in grasping of geometrical facts easily.
- It provides opportunities for exercise in reading scales, logarithmic tables, statistical tables, working with calculator, etc.
- It helps the students to develop proper skills to enable them to handle mechanical gadgets, e.g., calculators to make their work quick, easy and accurate
- It helps the students to develop the power of observation, analysis and conclusions.

Objectives of Mathematics Laboratory

- To provide readily accessible rich manipulative materials to emphasis on "learning by doing".
- To develop an attitude of enquiry.
- Remove the weakness of present day mathematics education.
- To develop much needed confidence in students.
- To generate interest in the subject.
- To make the students divergent thinkers.
- To make the children to look for pattern and ask questions.

Suggestions in the Use of Mathematics Laboratory

Organizing and conducting:

- ❖ There should be coordination between theoretical and practical work
- ❖ The purpose of practical work (experiment) should be made clear to the students and the pupils should be asked to keep a faithful record to what they do and observe.
- ❖ The practical work should be well graded according to the age and intelligence of the students.

Grouping of pupils: Individual work by the pupils in always preferred to group work. But if equipment and accommodation is inadequate, group plan should be adopted. In order to keep the normal social situation, the students should be free to choose their fellow students for the practical work

Preparation for individual and group work: There should be a place for everything and every thing should be to at its place. 'The equipment should be placed at a convenient place. For this, pupil assistants should be chosen in turn. This will solve the problem of laboratory assistants, and at the same time, the pupils will get the opportunity of handling and manipulating the apparatus.

Discipline: There should be no noise in the laboratory and the students should be kept busy during their work. A list of laboratory rules should be asked to follow them strictly.

Instructions to pupil: Students should be given instruction before they start with the practical work.

Pupil's practical notebook: All the pupils should maintain a notebook, which should preferably be plain.

Equipments in Mathematics Laboratory

Many of the equipments required in mathematics laboratory have been discussed while discussing laboratory method. However, a laboratory may contain the following equipments:

1. Charts and pictures: It should contain different types of charts and pictures concerning various topics and sub-topics of mathematics, charts and pictures showing application of mathematics in life and other fields of knowledge and history of mathematics. Few pictures

and photographs of the great mathematicians may also be hung on the walls of mathematics laboratory. Formula may also be demonstrated through charts.

2. Weighing and measuring instruments: Various types of weighing and measuring instruments like tape measures, balance stand weights, graduated cylinder, etc. should be placed in laboratory.

3. **Drawing instruments:** Wooden instruments for demonstration work of the teacher and small instruments within a small geometry box must also be available in the laboratory to learn geometry and mensuration.

4. Models: Models (which are the copies of real objects) of square, round and rectangular cardboard or thick paper, wooden or slope pieces to give the knowledge of numbers, numerals digits, area of four walls, cross-roads, circular figure, etc. should be available in the laboratory.

5. **Concrete materials:** Concrete material like breads, balls, sticks, pebbles, seeds, coins, match-box, wooden and card board pieces of different shape, disc for teaching fractional pans, scissors, pins, hammers, nails, thread ball, rope, thermometer, etc. should be placed in mathematics laboratory.

6. **Surveying instruments:** Some surveying instruments like:

(a) Angle Mirror - used for laying out right angles in the field.

(b) Sextant - used for finding out angles of elevation and depression, altitude of body and width of a river.

(c) Chinometers - used for measuring distance and heights of objects.

(d) Level-used in levelling the surfaces.

7. Some other instruments:

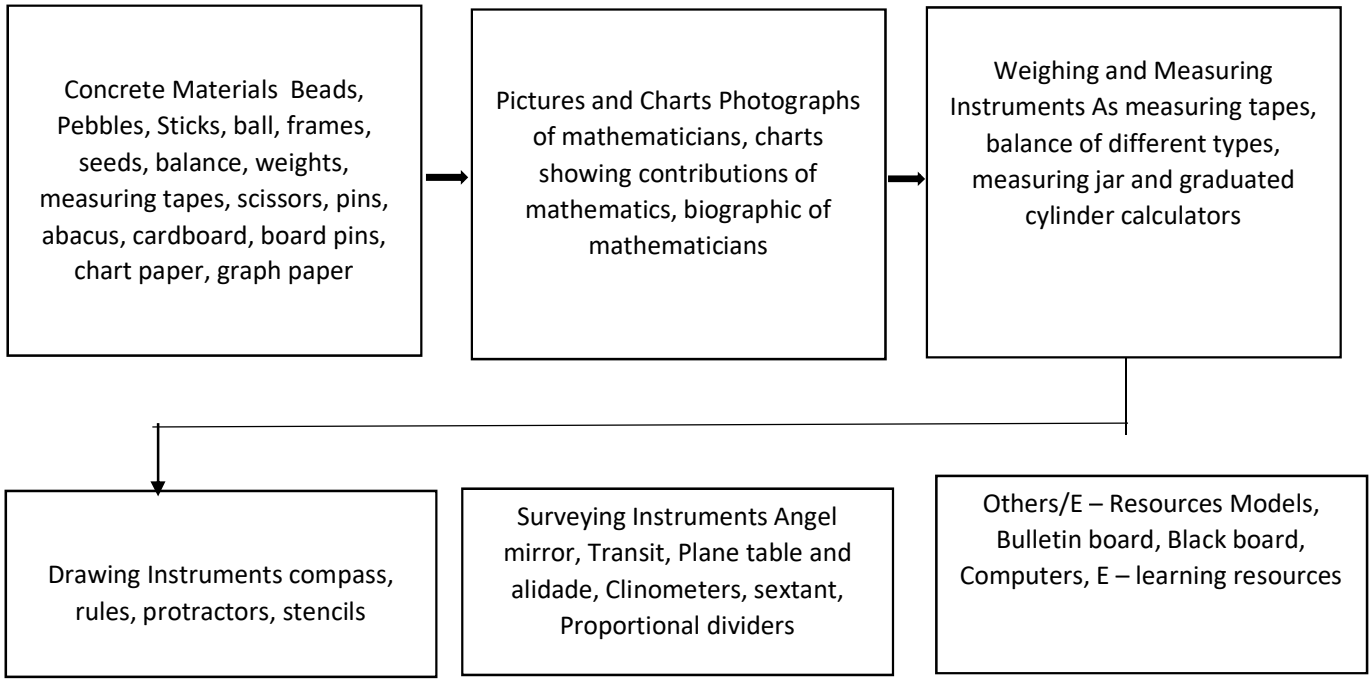
(a) **Proportional divisors** - used for enlarging or reducing the pictures, maps, diagrams, etc.

(b) **Slide rule** - used for multiplying and dividing the numbers.

(c) **Calculating machine** - a simple calculating machine has a number keyboard similar to that of a typewriter. Different numbers are keyed on it to get a final answer of addition, subtraction, division and multiplication.

(d) **Calculator** - used for earning addition, multiplication, subtraction, division, finding out the square roots of numbers, and storing any numbers. With this, we can find trigonometric and logarithmic value of any numbers; calculation becomes, easy, simple and accurate.

(e) **Computer** - a computer is a mechanical and electronic brain which is much more quick and powerful than the combined brains of hundreds of human beings. With the computer, very complicated problems can be solved very easily. Results are accurate and reliable.



Mathematics Club

The Present curriculum does not provide ample opportunities for the students for self-expression, independent thinking, constructive activities and other projects. There is a little scope for the students to solve the practical problem and no time is allowed for such type of work in the timetable. Through the present-day formal classroom teaching, there is a very small possibility of realizing the aims and objectives of teaching mathematics at junior as well as the senior secondary stage. Naturally, there arises the need for such an organization which can channelize students' energies towards desirable goals. Such an organization which caters for the inculcation of heuristic attitude, genuine interest in mathematics and mathematical activities, supplement the work of the classroom and puts the syllabus on a practical bias, may be names mathematics club.

Math club/forum is a group of individuals getting together to organize events, discuss and debate on various topics pertaining to mathematics. The club arranges various events such as birthdays of mathematicians, math days, etc. Also, the clubs and forums are engaged in organizing discussions, debates, seminars, study tours, etc. Ultimately, math clubs/forums help learners in developing interest and motivation in mathematics learning. There are different ways of involving learners in learning mathematics; math club/forums play a major role. So, as a math teacher, it is your duty to initiate processes to develop math clubs/forums. The math club/forum works under the guidance of the math teacher.

Significance of Mathematics Clubs

1. It inculcates heuristic attitude among the students.
2. It helps the students to develop their inventive faculties.
3. It helps the students in arousing and maintaining their interest.

4. Through mathematics club, the learning of mathematics becomes joyful.
5. It helps the students in the utilization of their leisure time profitability.
6. It helps the students in developing the habit of self-study.
7. It helps in bringing the school and society close together, as only through its activities, parents become familiar with various programmers of the school.
8. In mathematics club, the students can listen to great mathematicians.
9. In mathematics club, the students do their work together. They develop social qualities such as cooperation, tolerance and qualities of leadership.
10. It helps in satisfying more and more about the topic as in classroom. Due to shortage of time, teacher cannot discuss every topic in detail.
11. It helps in satisfying the instincts, urges, interests and needs of the gifted students.
12. As mathematics club provides motivation and incentive firm independent work, it helps in preparing future mathematician of the country.
13. It develops in children a sense of healthy competition for better cause.
14. It helps the students to widen their outlook and enable them to apply their knowledge of mathematics in certain life situation.
15. It develops the interest of the students in some mathematical hobbies like solving a puzzle, preparing charts, models, etc.
16. It helps the student to keep themselves in touch with the latest knowledge and development in mathematics and their effect on human life.
17. It develops keen use for observation in the students.
18. It develops the critical thinking and reasoning power of the students.

Objectives of Mathematics Club

The objectives of mathematics club can be summarized as under.

1. Mathematics club helps in the proper utilization of leisure time.
2. It helps in arousing and maintaining students' interest in mathematics.
3. It provides the students with opportunities to develop their explorative, creative and inventive faculties.
4. It inculcates the habit of self-study and independent work among the students.
5. It offers an ideal avenue for a free exchange of mathematical ideas and for frank and helpful criticism of these ideas.

6. It provides an informal and a social atmosphere, which the classroom can hardly provide.
7. It offers excellent opportunities for free consideration of matters of special interest to the members without the necessity of following any particular sequence as required in a classroom.
8. It helps in developing heuristic and problem solving attitude among the students.
9. It provides opportunities for students to translate the theory into practice and to apply their learning in daily life situations.
10. It helps the student to imbibe social qualities like co-operation, tolerance, adjustment and open-mindedness, as they work in groups.
11. It helps to meet individual needs of the students are every member gets an opportunity to work in his areas of interest.
12. The informal knowledge acquired through mathematics club activities supplements classroom learning.
13. Mathematics clubs extend learning beyond the limits of the classroom.
14. Mathematics clubs may meet during school hours, their activities may often extend to out of school hours at home in the laboratory, in the field etc.
15. Club activities provide first hand experiences to the learners as they participate in model making, arranging for exhibition, field work, laboratory work and so on.

Organization of Mathematics Club

Every mathematics club should have a constitution and every member should abide by it. The head of the institution should be the patron of the club and the teacher-in-charge the sponsor. The patron extends all types of facilities and cooperation for the successful execution of the club. The sponsor is just an advisor and helps in making the club self-conducting. He guides and guards the members and do not dictate. It is on him that the club can be kept in its high pitch of activity and interest. For this, he should be intellectually mature and educationally well informed.

There should be an Executive Body elected from amongst the students consisting of a chairman,, a secretary, an assistant secretary, a treasurer, a librarian, a shopkeeper, a publicity officer and class representative.

The Chairman usually presides over all the functions of the club. The Secretary maintains the record of all the activities of the club, carries out correspondence and invites guest. The

Assistant Secretary assists the Secretary and acts on behalf of him in his absence. The Treasure keeps up all the accounts. The Librarian carries out his own usual duties. The Storekeeper maintains the record of the equipment of the club. The Publicity Officer publicizes the club through different means.

The membership of the Mathematics club should not be imposed upon students and restricted. It should be voluntary and open to all the students in the school. A nominal fee may be charged from each member and the club should try to tap other sources.

To start with, some temporary officers may be selected from amongst the students. This committee should formulate the constitution of the club before the election of office bearers is held and the membership is thrown open. The constitution of the club should include decisions and directions about the following aspects:

i. The name of the club and its aims. It may be named after some renowned mathematicians.

ii. Condition and procedure of becoming member.

iii. Means to finance the club membership fee or other sources.

iv. Nature, place and timing of meetings to be held during the session. It is always good to hold the electing of the club twice or once in a month.

v. Types of activities to be taken.

vi. How the office-bearers are to be elected, what are their duties and how to fill the vacant post of office-bearer.

vii. Expelling of members.

Activities of Mathematics Club

1. Holding discussions, meetings, declamations, debates, paper reading contests, etc. on important topics of mathematics. Even if all the students do not participate in these activities at the school level which of course, will be the situation, these activities do them good in that they have an opportunity to listen to their class fellow and get inspiration from them.

2. Organization of inter-class or inter-school declamation contests, debates and symposium on important mathematical topics.

3. Collection of useful material of mathematics for mathematics laboratory

4. Drawing of charts, pictures, etc. and preparation of models.

5. Helping in the maintenance and organization of mathematics library and laboratory.

6. Arranging extensions lectures of renowned mathematicians on some important and useful topics.

7. Making arrangement for listening to the radio talk on some topic of mathematics.

8. Making arrangement for talking advantages of television talks and lessons concerning mathematics organizing competitions like mathematics.

9. Making arrangement for organizing competitions like mathematics games, riddles, puzzles, catch problems and quiz competitions.
10. Improving and preparing handmade apparatus.
11. Holding mathematics exhibitions and fairs annually.
12. Arranging excursions and visits to places of mathematical interest.
13. Every school should publish a magazine and the original contribution of students should find a place in it to serve as an incentive. At places where it is not a possible to bring a regular magazine, a wall magazine should be class wall magazines, which may be changed once a month. It will enable the students to have necessary motivation.
14. Making arrangement for the publication of newsletter and bulletin of the club.
15. Making arrangement for the guidance of those students who are poor in mathematics.
16. Providing opportunity to gifted students to show their talent.
17. Celebrating days and events of the history and men of mathematics.
18. Rendering all the possible services to the community.
19. Making arrangement to know about the working of calculator, computer, etc.

Therefore, there should be mathematics club up to senior secondary stage, which can provide a large number of activities, thereby widening and deepening the interests of the students and provide means of developing desirable ways of utilizing leisure time.

But we should not forget that the success of mathematics club depends on proper accommodation, necessary equipment, encouragement of the head of the institution, interest of the students and above all enthusiasm, resourcefulness, and ingenuity a mathematics teacher possesses.

Co-curricular (Recreational) Activities in Mathematics Clubs

Mathematics is considered to be a dry subject by many students. But it has its recreational value also. Be organizing recreational activities, the teacher can counteract the impression of the students, and at the same time, can remove the monotony of the classroom. Recreational activities are sort of leisure time co-curricular activities, which not only sharpen wit and stimulate reasoning and thinking power of the students, but also motivate the students and provide a lively and interesting picture of the subject of mathematics. There are many mathematical problems, games, mathematic numbers, riddles, magic square, etc. that bring aesthetic enjoyment, variety and develop taste for mathematics besides providing the opportunities to the students to appreciate the power and beauty of mathematics and mathematicians.

Every school should publish a magazine and the original contribution of students should find a place in it to serve as an incentive. At places where it is not a possible to bring a regular magazine, a wall magazine should be class wall magazines, which may be changed once a month. It will enable the students to have necessary motivation.

Practice Questions

1. How will you organize a mathematics club in your school? Suggest some of the activities for this club.

2. What is the utility of mathematics club?

3. Discuss the role of mathematics teacher in organization of mathematics club in the school.

4. Comment on need, importance, establishment and activities of mathematical club.

5. Justify the need of a Mathematics Laboratory for effective teaching of mathematics.

6. Write note on Mathematics Laboratory.

7. Discuss the role of mathematics library and mathematics laboratory in the teaching of mathematics at the secondary school stage.

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Learning Resources: Textbook

Introduction

Any material, which works as a medium in the process of learning and teaching, is called textbooks Bacon (1935) has defined textbook as "a book designed for classroom use, carefully prepared by experts in the field and equipped with the usual teaching devices" However, the term textbook, here broadly means the material employed by students and teachers as standard work on a subject of mathematics

Modern-day textbooks of mathematics have the following flaws.

1. In many cases, subject-matter is faulty

2. Distribution of topics is faulty

3. These suffer from an excess of logic and a dearth of psychology

4. Textbooks are written on deductive lines.

5. Present textbooks use artificial instructional settings. These emphasize telling and showing as the only means of instructions.

6. Present textbooks do little to develop real learning situations.

7. There is no correlation of one branch of mathematics with other branch and also with daily life

8. These textbooks follow a rigid step-by-step grade placement of the topics.

9. Generally, textbooks are not revised years together.

10. No guidelines are given for the teachers.

Need/Purpose/Values of Textbook

Textbook in mathematics enjoys a unique position in the field of education. Textbooks represent a careful selection of material, which have been organized for instructional purposes. Following are the needs of textbooks:

Use of Textbook for the Teachers

1. Textbook is a valuable guide and generally written by experienced teachers. It serves as a reference book or teacher, a reliable source of basic material and information for the teacher.
2. It helps teacher to organize material of learning and instruction in a well-organized and systematic manner
3. It contains relevant matter and guidelines concerning the syllabus of that class. Therefore, it guides the teacher about the boundaries of his teaching for that class.
4. It helps the teacher in planning his lessons, deciding about the methods of teaching as a particular topic and preparing relevant aids.

Dr. (Mrs.) Megha D. Gokhe
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5. As it contains few solved examples on every topic, it helps the teacher to become familiar with the ways and methods of attacking different types of problems in the class.

6. As it contains so many solved and unsolved problems, it saves the time of teacher which otherwise would have been wasted in setting new problems.

7. It helps the teacher to give practice and assign homework to the students according to their individual differences.

8. It helps to plan sequential relationships of material, and suggests possibilities of correlation and related projects.

9. It gives correct answers to all problems. The teacher comes to know that his solutions are correct.

Use of Textbook for the Students

1. It contains hints for the solution of some difficult problems and answers to all the problems given in the exercises, which are very helpful for the teachers and students.
2. It helps the students to know what they are supposed to learn in a particular class
3. It helps the students in supplementing and correcting the class notes.
4. It saves the time, money and energy of the students.
5. It presents application of principles and factual materials.
6. It serves as cheapest and most reliable source of information for the students.
7. It helps the students to develop the habit of independent self-study
8. It meets the needs of all the students as it contains work for all categories of students, 1.e., background, average and gifted.

9. It helps the students to study a topic in advance.

10. It also help the students as one of the means of reviewing and recognizing his knowledge.

11. In our classroom instruction, textbook is a concise source of material for review.

12. In lower classes, textbooks in mathematics with coloured illustrations provide an incentive to learning and provide attraction for the young students.

13. It is the connecting source of pre-lesson and post-lesson learning for the students.

14. Textbooks in mathematics promote periodical learning, relearning and reinforcement in addition to formal classroom learning.

15. If by chance a student fails to attend the school for a few days, he can make up his deficiency with the help of textbooks and catch up with the class.

Use of Textbooks for Maintaining uniformity of Standard

1. It helps to present materials basic to a general understanding of a course.

Unit 5 (B): Learning Resources Textboook

2. The basic task of textbooks is the presentation of data, from the subject area such as mathematical data, the basic items agreed upon as facts by the community of scholars in the field of mathematics.
3. It helps in maintaining uniformity of standard between sections of some particular class, between different teachers, who are teaching some particular class, between different schools of the tensile, district, state and nation (take the case of mathematics book written for 10+2 system of education), having same curriculum.
4. Conducting of public examination will also be possible when there will be uniformity in textbooks.
5. Textbooks in mathematics decide for the teacher, his teaching, for the student, his learning and for the examiner, his task.

Characteristics of Good Textbook

1. Textbooks should be appropriate to the age level of the students.
2. Content of the mathematics textbooks should be correlated with the life activities.
3. Textbooks should have illustrations, diagrams and sketches at the appropriate places. lt should be done with the consultation of an artist.
4. Style of writing of textbooks should be simple.
5. Textbooks should be free from mistake.
6. Textbooks should have sufficient material to motivate the students to solve problem
7. Topics of the textbook should be arranged from simple to complex
8. Subject-matter should be arranged on the principle of concentric order so that topic being taught may be correlated with the past and future learning
9. The principle of learning by doing and from concrete to abstract should be followed for the illustration of subject-matter
10. Textbook should be in accordance with the objective set for the teaching of mathematics because textbook is means and not the end.
11. There must be ample properly graded textbooks, i.e. these must be discriminative for bright, average and dull students.

Dr. (Mrs.) Megha D. Gokhe
Principal, TSCER

12. There should be provisions for individual differences.

13. Textbook must have the provision for slow learners and backward ones.

14. There must be accuracy of facts and figures in the textbook

15. Textbook must have satisfactory data.

16. There should be no language Problems.

17. Textbook must have satisfactory data

18. It must contain short answer and objective type questions in addition to essay type questions.

19. There must be historical and biographical notes concerning the topics and also of great mathematicians.

20. Textbooks must be written according to the scheme laid down in the syllabus.

21. Generalizations must be made effectively whenever possible.

22. To avoid spoon-feeding, there must be too many solved examples

23. It should be up-to-date as regards the subject-matter

24. To avoid confusion, the symbols and terms used must be those which are popular and internationally accepted.

25. Author must have appropriate qualification and experiences so that their experience benefit both the teachers taught

26. There must be provisions for the use of inductive, analytic. project and laboratory methods

27. Selection, gradation, presentation and repetitions of matter should be in consonance with the nature of subject of mathematics

28. The syllabus must be divided into units or sub-units with meaningful and interesting headings

29. There must be suggestions for correlation. project work assignment work and fieldwork

30. It should also satisfy the needs of the students from examination point of view

31. Get-up of the textbooks should be attractive, i.e., quality of the paper, quality of the printing and binding, etc should be superior

32. Textbooks must be readily available in the market

33. Price of the textbooks must be reasonable

34. The methods of presentation should follow in view of the learning process and the latest information about the subject

35. Problems in the textbook should be related to life situations.

36. It must contain correct answers to all problems at the end of a chapter or section of the book

How to Use the Textbooks?

Teacher must note that textbook is a means and not an end in itself it should never set a limit. Textbook is an aid in teaching, not a substitute for teaching. He should not have the opinion that his work is only to transfer the material of the textbooks in students head Teacher should not use the textbook as the only source of instructional material

One important point, which the teacher should keep in his mind about the use of instructional textbook is that he should not blindly follow the textbook, but intelligently and carefully adopt s to his use. Textbooks is not our master to be feared, but it is our servant to assist us. Teacher can make lest use of textbook, if he supplements it by reference readings, by his oral exposition, and other illustrative and objective techniques, which he has learnt through his experiences.

Critical Analyses (How to Select a Good Textbook?)

The main purpose of evaluating the textbook is selection, improvement and research. Evaluation of textbooks takes into account the objectivity of different aspects of textbooks, needs of the learner, requirement, interpretativeness of the criteria and effectiveness in teaching-learning situations.

Unit 5 (B): Learning Resources Textboook

Generally the tools that are used in evaluation of a textbook are report card, scoring card scoring sheets, profile, observations schedule, analysis sheets, evaluation perform, questionnaire checklist and rating scale

Evaluation of textbook is a cooperative endeavor It can neither be left to the opinion of experts nor can it be left to the decision taken on the basis of the liking or disliking of teachers or taught (student) The panel of experts, through the process of try out, pooling the opinions of teachers, supervisors and pupils can evaluate it

It has been observed that the procedure followed by State Education Department for evaluation of textbook have a number of defects. Firstly, the same review form, scorecard is used for all grades which leads to defective assessment because the objectives, choice of content, etc differs from grade to grade Secondly, due to lack of specializations in theses form cards, only vague opinions can be expressed such as style is defective or printing is proper, etc. It is not possible to indicate on those form cards certain qualitative aspects of textbooks. Lastly, arbitrary weightage is given to various aspects of textbook evaluation, e.g., conformity to syllabus may be assigned 5 marks, while emphasis on organization, language, etc may be assigned 10 marks each. This type of weightage is subjective and not reliable.

Below is given one evaluation scale for evaluating the textbooks in mathematics The scale contains 47 statements distributed over eight aspects. Textbook is evaluated on the basis of five points, i.e.,

Aspect Statements	5	4	3	2	1
I. Physical Aspects					
The size of the textbook is quite adequate to the level of the pupils.					
The type of syllabus is adequate to the age level of the pupils.					
The textbook is revised from time to time					
The book is printed by authentic publishers.					
The binding of the hook is durable					
The paper used is durable, attractive and facilitates reading					
Proportionate Margin is left for students to note the important points.					
II. General Nature					
Experience and qualification of the author are up to the marks.					
The objectives are clearly defined in preparing the book.					
The table of content is presented attractively and comprehensively					
A list of illustration is given in the book					
Answers are accurate and correct.					
III. The Nature of Content					
The material presented is up-to-date.					
Style of writing is simple and within the level of understanding of pupils					
The material given in the book confirm to the prescribed syllabus					
IV. Organization					
The chapters are divided chronologically.					

Dr. (Mrs.) Megha D. Gokhe
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Unit 5 (B): Learning Resources Textboook

All the topics of syllabus are given adequate and appropriate weightage					
Content is correlated with other branches of mathematics and with other subjects					
The textbook provides for individual differences in terms of interest, needs and abilities					
The generalization are supported by facts.					
There are provisions within the textbook for drill and review					
V. Illustration					
The space denoted to illustration is in proportion to the demand of the grade level.					
There is use of visual aids, charts, graphs, diagrams and pictures to give the meaning					
The depiction of illustration is clear and accurate					
The illustrations supplement the information.					
VI. References					
References for pupils and teachers are given.					
A bibliography is given at the end.					
VII. Style					
The expression is clear, logical and simple.					
Symbols used in the book are internationally accepted					
Graded exercises for weak, average and gifted students are presented					
Not too much examples are given.					
The vocabulary norms are satisfactory.					
Practical application is emphasized.					
Problems are related to the life situations.					
VIII. Instructional Aids					
A list of individual and group activities is given.					
Directions for further study are given.					
Exercises contribute to problem-solving and other skills.					
Suggestions for using instructional aids are given.					
Exercises serve the need and interest of the pupils.					
IX. General					
On the whole, textbook is accepted and appreciated by the reviewer.					
The book is written with progressive outlook.					
The price is reasonable.					
It helps in achieving the aims and objectives of teaching mathematics.					
There is provision for the use of inductive, analytic, laboratory, heuristic and project methods.					

Practice Questions

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Unit 5 (B): Learning Resources Textboook

- 1. What are the functions of textbook in mathematics? How would you select a good textbook for class use and how would you use the same?
- 2. How would you evaluate a suitable textbook in mathematics for your students?
- 3. What are the characteristics of a good textbook in arithmetic? On the basis of these characteristics. evaluate any textbook in arithmetic, which you have consulted during your teaching practice
- 4. Explain the importance and qualities of good textbook in teaching of mathematics
- 5. Why is the textbook in mathematics indispensable for school students? What considerations are kept in mind for selecting a textbook in geometry?
- 6. What is textbook? State the qualities of a good textbook. What are the considerations to be kept in mind while selecting textbook?
- 7. Discuss the characteristics of a good textbook in mathematics. What improvement will you suggest in the mathematics textbook at middle class level of your state?

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Digital Resources for Teaching Mathematics

INTRODUCTION

The term ‘digital learning resource’ is used here to refer to materials included in the context of a course that support the learner’s achievement of the described learning goals. A digital resource is self-contained, reusable, and may be portable between learning systems.

Need of ICT In Mathematics Teaching

- Visualization of teaching materials facilitates understanding in mathematics and the use of digital resources facilitates appointed visualization process.
- Digital resources the efficiency of mathematical thought, enables learners to make conjectures and immediately tests them in non-threatening environment.
- Digital resources also offer multiple mathematical representations that enhance generality of mathematical concepts, and provide opportunities for counter-examples, unlike in paper-and-pencil environments.
- Digital resources also enhance the curiosity that may drive inventions as illustrated in computational mathematics.

GeoGebra

GeoGebra is a simple but powerful tool which can be used by students to understand maths concepts, help solve problems and check solutions. GeoGebra can be used for statistics, probability, geometry, trigonometry, algebra and functions.

Its creator, Markus Hohenwarter, started the project in 2001 as part of his Master's thesis, at the University of Salzburg, continuing it at Florida Atlantic University (2006-2008), Florida State University (2008-2009), and now at the University of Linz together with the help of open-source developers and translators all over the world.

Geogebra is available on multiple platforms with its desktop applications for Windows, MacOS and Linux, as a tablet app for Android, iPad and Windows, and as a web application based on HTML5 technology.

Constructions can be made with points, vectors, segments, lines, polygons, conic sections, inequalities, implicit polynomials and functions. All of them can

be changed dynamically afterwards Elements can be entered and modified directly via mouse and touch, or through the "Input bar" GeoGebra has the ability to use variables for numbers, vectors and points, to find derivatives and integrals of functions. Teachers and students can use GeoGebra to make conjectures and to understand mathematical topics.

Advantages of Applications of GeoGebra

- GeoGebra makes learning an abstract concept much more meaningful.
- It visualizes related concepts and how they affect each other.
- It allows everyone equal access to an outstanding learning tool.
- The price is right.
- Anyone can use applets created with GeoGebra without restriction.
- Anyone can use this tool to create applets.
- Conceptual understanding is enhanced.

Limitations of GeoGebra

- Students with no previous programming experience will be hard to manage with inputting algebraic commands via interface input field. Although it is not difficult to learn the basic commands of GeoGebra, some students may feel uncomfortable or completely lost.
- Some methodological approaches (for example, independent "research" and experimentation) may be inadequate for many students. Without proper guidance and explanations by the teacher, many students who did not understand the material on a regular teaching and do not have the necessary knowledge for such work will make unarticulated, ineffective and meaningless actions.
- In technical terms, GeoGebra has no built-in support for animation and 3D display. Technical upgrade with GeoGebra module for animation and 3D display should become an important element of future versions of this package.

Future enlargement of GeoGebra software will certainly include major symbolic features of the current CAS, which will increase the possible complex application of GeoGebra in mathematical analysis field. There are small number of scientific studies on the impact of GeoGebra in teaching and learning of

mathematics. In other words, this tool extends the concept of dynamic geometry on the fields of algebra and calculus.

VIRTUAL MANIPULATIVE

As a result of innovations in technology, the prevalence of the Internet, and the increasing availability of computers in classrooms and homes, an enhanced approach for teaching and learning mathematics using manipulatives and computers is emerging. This new approach essentially creates a new class of manipulatives, called virtual manipulatives, as well as new capabilities, or toolkits, for computer programs that use visual representations. These new virtual manipulatives have all the useful properties of existing computer manipulatives while overcoming many of their disadvantages, yet very little is known or written about them. The purpose of this article is to establish a working definition of virtual manipulatives, highlight examples of virtual manipulatives on the Internet, and discuss their current and potential classroom use.

Although many people currently use the term virtual manipulative to refer to any computer-generated image that appears on a monitor and is intended to represent concrete manipulatives, this meaning is much too simplistic to be descriptive or useful. The ability to manipulate the visual representation, or object, on the computer connects the user with the real teaching and learning power of virtual manipulatives, i.e., the opportunity to make meaning and see relationships as a result of one's own actions. This kind of user engagement distinguishes virtual manipulative sites from those sites where the act of pointing and clicking results in the computer's providing an answer in visual or symbolic form.

But, the term virtual manipulative should be restricted to describing this interactive capability. A virtual manipulative is best defined as an interactive, web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge. Currently, virtual manipulatives are modelled on the concrete manipulatives commonly used in schools, such as pattern blocks, tangrams, fraction bars, geoboards and geometric solids. When such objects are available through the Web, they may be considered virtual. However, their ability to be used interactively-i.e., to allow the user to engage and control the physical actions of these objects combined with the opportunities that they offer to discover and construct mathematical principles and relationships, distinguishes them as virtual manipulatives With current technology, the most common way of effecting these movements on a monitor is by using a computer mouse to point, click and drag. The future will

undoubtedly bring many other ways of moving virtual manipulatives, perhaps through such features as voice commands and infrared signals.

Types of Virtual Manipulatives

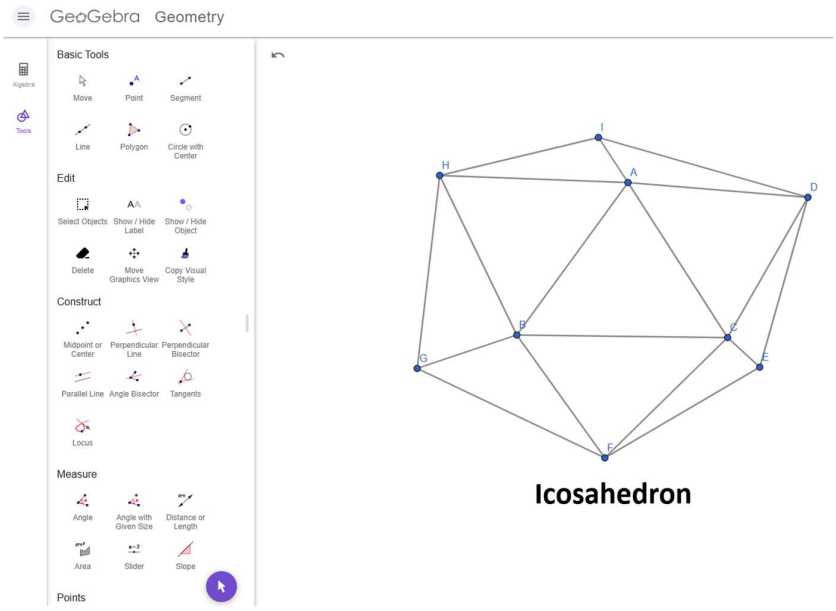
Two types of virtual manipulatives are available as teaching and learning tools. They are:

- Static Visual Representation
- Dynamic Visual Representation

Static visual representations are essentially pictures They ordinarily associated with pictures in books, drawings on an overhead projector, sketches on a are the sorts of visual images chalkboard, and so on. Although such representations resemble concrete manipulatives, they cannot be used in the same ways that concrete manipulatives can i.e., a student can actually slide, flip and turn concrete manipulatives but cannot not perform the same actions with a static picture of the concrete manipulative. These static visual representations are not true virtual manipulatives.

In contrast, dynamic visual representations of concrete manipulatives are essentially "objects" They are visual images on the computer that are just like pictures in books, drawings on an overhead projector, sketches on a chalkboard, and so on In addition, these dynamic visual representations can be manipulated in the same way that a concrete manipulative can.

Just as a student can slide, flip and turn a concrete manipulative by hand, he or she can use a computer mouse to actually slide, flip and turn the dynamic visual representation as if it were a three-dimensional object This kind of visual representation is truly a virtual manipulative.



The above figure is an example of a tab on a virtual manipulative resource. Here, students can select new 3D shape and rotate it by using joystick or mouse, and complete the task of analyzing the shape in terms of edges, vertical and faces. The student can also mark the counted/observed faces by different colors.

Advantages and Applications of Using Virtual Manipulative

1. It teaches underlying values and skills. Problem-solving, patience and attention span, critical thinking, creativity, concentration, etc.
2. Students feel more capable and competent. Because they do things on their own and discover things on their own, they feel less dependent to their teachers.
3. They will see real-life applications of concepts. Rather than teaching them concepts, manipulatives allow them to literally grasp each situation and they will feel the relevance of the concepts.
4. They can keep the students occupied, attract a lot of attention, and can keep it as long as it is developmentally appropriate.
5. Easier for students to understand and reflect on the topic. Since everything happened under their control, it is easier for them to analyze what they did and it allows them to "play around the concepts".
6. Student can save work for future reference.
7. The work done can be linked to other resources like .doc, .pdf, .xls.
8. Work can be accessed anywhere there is a computer.

9. Formatting is easy (potential to alter, add color, etc.).
10. A way to link students to the virtual world, virtual manipulatives will improve in the future

Limitations of Using Virtual Manipulative

1. Many teachers are unaware of the capabilities of virtual manipulatives and do not currently use them in lessons during regular mathematics instruction.
2. Limited technological resources are a highly limiting factor in the use of virtual manipulatives. In order for them to truly display their learning and cognitive benefits, they must be used by every students and not in a mere teacher demonstration. Therefore, this lack of available technology is a true disadvantage to students access to virtual manipulatives.
3. Virtual manipulatives are also not designed in a way to support collaborative learning which is an especially large problem for the socio-cultural constructivist educators.
4. Employing manipulatives in a class is not straightforward, and good employment requires carefully defining the role of the teacher, and the aims and the potential tasks involved. This could require a lot of work on the part of the educator making several stray from using virtual manipulatives altogether.

Despite the disadvantages listed here, it is safe to say that if one can overcome the initial hesitation to the introduction of technology, the advantages of virtual manipulatives far outweigh any disadvantages.

PRACTICE QUESTIONS

- What is digital resources? How can you integrate it in your mathematics lessons?
- Explain the use GeoGebra in teaching mathematics. Also discuss its advantages and disadvantages.
- Explain the concept of virtual manipulatives while referring to its advantages and disadvantages.

Competencies of Mathematics Teacher

INTRODUCTION

A teacher is a decision-maker. Teachers make judgements pertaining to objectives, learning activities, evaluation procedures as well as quality of the classroom environment. To be a teacher means to be a decision-maker. Within an open-ended arena, problems arise and solutions need to be found.

Importance of a Teacher in Mathematics

1. The first and most obvious role of a mathematics teacher is that of initiator, director and evaluator of learning experiences. Presumably, this is the chief reasons why teachers are employed to see that learning takes place.
2. Teacher is the representative of society or a person who is concerned with transmitting the values and standard of culture and community. By perception and examples, he tries to develop the moral attitudes, thinking patterns, the life goals among students, which he feels, are the signs for good citizens living a good life. In this role, he is more or less faithful mirror of the society in which he lives.
3. Like other subjects, in teaching of mathematics also, many things are planned and set in advance. Curriculum and syllabus are framed, new techniques and devices are explored, variety of audio-visual aids are collected, appropriate textbooks are written, but all these things have no use if there is no competent and effective mathematics teacher as it is teacher who always holds the key position in the process of teaching and learning. In other words, he may be considered as an analyst because many changes occurs because of him and would not occur if he was absent.
4. Various eminent educationists have highlighted the role of teacher and his importance. Some of them are given below:

"*Teacher should be filled with a modernist-attitude, with progressive, with a forward looking direction. Unless they themselves have it, they cannot make their students forward looking.“*
-Sarvapalli Radhakrishnan

"A teacher has to help in the transmitting of higher values to his pupils, through his personality and through the goods of culture which are his instruments. A teacher has to help the bud into full bloom and not to make paper flowers to

satisfy his whim. The growth of a morally autonomous personality is the aim and of his endeavor.“ - Jawahar Lal Nehru

"A teacher can never truly unless he is still learning himself. A lamp can never light another lamp unless it continues to burn its own flame. The teacher who has come to the end of his subject, who has no living traffic with his knowledge but merely repeats his lesson to his students, can only load their minds, he cannot quicken them. Truth not only must inform but also inspire. If the inspiration dies out and the information only accumulates, then truth loses its infinity. The greater part of our learning in the schools has been a waste because for most of our teacher, their subjects are like dead specimens of once living things with which they have learned acquaintance, but no communication of life and love.“ -Rabindranath Tagore

5. Commenting on the importance of a teacher, American Commission on Teacher Education says, the quality of a nation depends upon the quality of its citizen, the quality of its citizen depends not exclusively, but in critical measure upon the quality of their education. The quality of their education depends, more than any other single factor, upon the quality of their teacher." Therefore, from this, we can conclude that quality of mathematics education to a great extent depends on the quality of a mathematics teacher.

In the following pages, let us discuss the qualities of mathematics teacher for the effective realization of the aims and objectives of mathematics teaching in our schools.

His qualities may be classified into two categories: (i) general qualities that are expected in every teacher and (ii) special qualities that are expected from a good mathematics teacher.

General Qualities of Mathematics Teacher

1. The teacher should be a guide, a helper and a friend.
2. The teacher must study the child, must know the effect of the environment on the child and should know the laws of learning for which a study of psychology is essential. He should adjust his teaching accordingly.
3. The sponsoring of co-curricular activities, participation in counselling and guidance, keeping careful records, maintenance of register work, realization of dues, making necessary reports, participation in worthy community interest are but a few illustrations of the range of demands upon the teacher.

4. His first and foremost obligation is to teach effectively. He must have the knowledge of various teaching methods so that he is able to communicate well.
5. He must have satisfactory mental health and adequate adjustment.
6. He should also be equipped with qualities like patience, love, respect for his profession, good character, leadership qualities like patience, love, respect for his profession, good character, leadership qualities, self-criticism and self-confidence.
7. Teacher should cooperate with parents to improve school and community relations and opportunities. There should be arranged meetings of social or civic relations and opportunities. There should be arranged meetings of social or civic groups, parents‘ association and old students’ association. He should explain the parents and visitors the facilities offered by school for teaching of various subjects.
8. He must have progressive and dynamic outlook, and well integrated and effective personality.
9. For successful classroom teaching, he must be familiar with the various factors governing attention, interest of the students and mental level of the children.
10. He should understand and motivate the children.
11. He must know that there are individual differences and therefore he must deal the children according to their capacities.
12. He must possess the habits of self-study.
13. A teacher must be sympathetic, affectionate and impartial in his outlook. He should have no favorites and should not have any prejudice or biased viewpoint for any of his pupils. His treatment should be fair to all, especially in making the papers and doing other evaluation about the child. There should not be any subjectivity and he should give no room for partiality. He must have neat and clean habits such as patience, confidence and hard work.
14. He must perform the role of disciplinarian. In other words, he must see that students remain within limits set up by society.
15. He must be a good artist in human relation a person who works with variety of techniques and produces situation that will stimulate learning.

He should sense the feelings and needs of the group and must know when to end the discussion of a given subject, when to change pace, and how to introduce topic.

16. He should help the students to understand and accept each other to work together to share material and experience.
17. He must also be a group builder who can know whether students knowingly or unknowingly block the progress of learning and what are those factors within group that can be used to stimulate learning.
18. He must perform the role of mental hygiene worker in reducing the anxiety of the students and in arousing their normal anxiety. He must help the children in meeting their psychological needs.
19. He must function as a referred agent to the counsellor. He must give reassurance and emotional supports to the students.
20. Teacher must perform the role of a social services worker, i.e., he must be offered an opportunity to help others in order to build a better society. He should lend a helping hand to the illiterate neighbours and farmers of the village in which the school is situated.
21. Still another role of the teacher is that of a parent figure. Children tend to look upon teacher as a substitute of parents.
22. Teacher must have adequate expression power both verbal as well as written and general knowledge.
23. Teacher should attend seminars and workshops with an open mind to learn. This is necessary for the teachers to keep themselves up-to-date in respect of subject-matter and teaching techniques.
24. The teacher must love his subject as well as his students. He must accept the students warmly.
25. We no longer accept the fact that the teacher is one who "teaches". Today, a true teacher is one who learns and inspires. Effective learning is possible when the teacher possess certain competencies to make learning simple and interesting.

Specific Qualities of Mathematics Teacher

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Unit 6 (A): Competencies of Mathematics Teacher.

Classroom teachers need ample opportunities to grow, develop mature and change. Staying in the classroom continuously, experiencing the grow, and change having few opportunities to interact with other professionals, attending few or no meeting at home or away, and lacking motivation does not encourage meaning and interest in professional teaching. How then the teaching of mathematics truly becomes a profession? Mathematics teacher should possess the following qualities:

1. Level of teacher's knowledge must be higher than that of the knowledge he is expected to impart. Mathematics teacher must have mastery over the subjects in order to become an effective teacher. mathematics with graduate or postgraduate degree.
2. A mathematics teacher should have undergone a proper teacher training course so that he may be able to teach mathematics effectively, and in such a way, that the aims and objectives of teaching mathematics may be properly realized.
3. He must be familiar with the aims and objectives of teaching mathematics at all levels of education. This will help him in the selection of subject-matter and methods of teaching.
4. He must have adequate command of instructional material in mathematics and the ability to face the class with confidence.
5. In addition to his knowledge of mathematics, he must have background knowledge of other fields as well. For example, physical and biological sciences, language, etc., this knowledge of other subjects will give the teacher a broader outlook and help him in developing confidence which is essential for an efficient discharge of his duties. It will help the teacher in correlating mathematics with other facts and subjects, and to make this subject meaningful and interesting.
6. In order to enrich his knowledge, the teacher must study mathematical journals and other books of professional interest. He must visit the schools of outstanding merits to see the quality and nature of work done and to have discussion with reputed teachers of the subject. He may become a member of some mathematical society.
7. Teacher must do professional research work and must write professional articles.
8. He must attend in-service programme in order to refresh his knowledge of the subjects.

Dr. (Mrs.) Megha D. Gokhe
Principal, TSCER

9. Teacher must be able to make his students familiar about the practical use of mathematics in day-to-day life problems.
10. Mathematics teacher must have knowledge of the history of mathematics and contributions and life history of great mathematicians.
11. He must possess the essential mathematical skills, e.g., problem solving skill, skill of computation, drawing and sketching, and working with calculator and computers.
12. He must possess an essential zeal and positive attitude towards the subjects of mathematics.
13. He must be able to prepare, administer and score the different achievement tests in mathematics.
14. He must be able to organize mathematics club in the school. He should also organize some fairs and exhibitions so that some items of mathematical interest are included. He should explain to the parents and visitors the part played by the students of mathematics.
15. He must be able to diagnose and provide remedial teaching for those students who are backward in the subject of mathematics.
16. He must motivate, insure and guide the gifted children.
17. He must try his level best to inculcate proper interest among the students in the subject of mathematics.
18. He should be able to cultivate originality, creativity and ingenuity among his students for the solution of mathematics problem. For this, he must be able to suggest alternative to certain procedures or rules. Only then he will be able to develop original thinking among students.
19. He must be able to assign suitable homework to his students according to their mental level.
20. He must be resourceful. He should be very quick and prompt in mental operations. He should frame new questions on the spur of the moment, and whenever the lesson is growing dull, he should create new exercise to maintain the interest of the students immediately.
21. He must be exact, accurate, reasonable and very systematic as mathematics is such a subject, which requires reasoning at each and every step. He must provide arguments for every step he takes. He must prepare his lessons well in advance.

22. He should always be on the lookout for new applications of old principles.
23. He must know about the availability of different audio-visual aids in teaching of mathematics and their proper utilization.
24. A mathematics teacher should be very frank. If he has made some mistakes in his calculations, he should at once admit his ignorance without any hesitation.
25. He must develop heuristic attitude among children. But in actual practice, very rarely is the student allowed to put questions to the teacher. Teacher's ignorance, incompetence, conceit or indifference is mainly responsible for this state of affairs. No mathematics teaching can be efficient, if he does not encourage and invite pupils to put him questions and treat them with sympathetic attitude.
26. Classroom teachers of mathematics should activity be involved in research. In conducting research, the teacher with the guidance of research specialist, needs to identify an experimental group as well as control group. The teacher should emphasize practical research.
27. Each school should have a professional library for teachers. The library needs to house teacher education textbooks, educational journals, audio-visual aids, pamphlets and other materials on the teaching of mathematics. These should be available to teachers to do professional reading in mathematics and to improve the curriculum. Teachers should be given special time during the day to read and reflect open what has been read. Ample opportunities need to be given to teachers to discuss about the content read by them.
28. Teachers should attend professional meetings in the school and by way of in-service education. An end result should be improvement in teaching performance in mathematics. Teacher should have ample opportunities and time to discuss ideas acquired with other professionals.
29. Administrators and supervisors must schedule an adequate number of faculty meeting during a school year and each faculty meeting needs to pinpoint selected problems identified by the teachers. Mathematics teacher must participate in faculty meetings.
30. Mathematics teacher must take university courses to continue professional education beyond the acquired degree. These courses should place heavy emphasis upon content and methods of teaching.

31. Teachers must attend workshops. It will be the opportunity to develop well professionally.
32. Teachers must attend in-service education programme. The plan of this programme must stress on how to motivate students and how to guide them to increase problem solving skill.

Lastly, it is the zeal, enthusiasm, sincerity towards work and devotion to his profession, which matters the most. The most effective mathematics teachers are those, who are able to grow not only in knowledge of their subject but in their understanding of life both in and out of classroom. For them, the classroom is fascinating laboratory of life wherein they grow in their ability to understand more about their subject, more about children and more about themselves as teacher and as individuals

Concerns for a Mathematics Teacher

While all curriculum areas share some of the same issues and concerns, individual curriculum areas seem to also have concerns specific to them and their courses. Following are few concerns for mathematics teachers:

1. **Dependent on previous knowledge**: As reader have studied about hierarchical nature of mathematics in earlier chapters, curriculum of mathematics builds on information learned in previous years. In case of absence of required previous knowledge among few students, a mathematics teacher is left with a dilemma of choice between remediation or forging ahead and covering material the students might not understand.
2. **Connections to real life**: There is a huge gap in theory and daily life practices related to mathematics. Nobody applies algebraic identities in solving mathematical problem of daily lives, although consumer mathematics (profit, loss, interest, etc.) is easily connected to daily life. However, it can often be hard for students to see the connection between their lives and trigonometry, polynomials and even basic algebra. When students do not see why they have to learn a topic, this impacts adversely on their motivation and retention.
3. **Cheating issues**: Unlike social sciences and language where students have to write essays or create detailed reports, math is often reduced to solving problems. It can be difficult for a math teacher to determine if students are cheating. Typically, math teachers use wrong answers and incorrect solving methods to determine if students did, in fact, cheat.

4. **Students with mathematical anxiety**: Some students over the time due to consecutive failure or boring methods of teaching make a belief about themselves that they are "not good at math" or "beyond my ability". They become anxious in mathematics class and during tests. Tackling this self-esteem related issue can be difficult indeed.
5. **Monotonous instruction**: The teaching of mathematics does not lend itself to a great amount of varied instruction. While teachers can have students' present material, work in small groups for certain topics and create multimedia projects dealing with math, the norm of a math classroom is direct instruction followed by a period of solving problems.
6. **Dealing with absenteeism**: When a student skips or misses a mathematics class at key instructional points, it can be difficult for students to catch up. For example, if a student is absent on the first few days when a new topic is being discussed and explained, a teacher will be faced with the issue of helping that student learn the material on their own:
7. **Grading concerns**: Mathematics teachers need to keep up with the daily evaluation of assignments. It does not give much help to the teacher if only final examination scores or grades are used for evaluation. Only by checking what mistakes students have made and working towards correcting those, the teachers will be able to use that information effectively.
8. **Need for after-school tutoring**: It is common for mathematics teachers to receive more demands from students who are requesting extra help. This requires a greater dedication on their part in many ways to help these students understand and master the topics being learned.
9. **Having students of different ability group in class**: A group of students in a classroom is always heterogeneous in nature with respect to their abilities. This might result from gaps in previous knowledge or each student's self-perception about his own ability to learn mathematics. Teachers must decide how to meet the needs of the individual students in their classrooms.
10. **Homework issues**: Curriculum of mathematics requires daily practice of learnt concepts. Therefore, the completion of daily homework assignments is essential for learning the material. Students who do not complete their homework or who copy from other students often struggle

at test time. Dealing with this issue is often very difficult for mathematics teachers.

Practice Questions

1. Explain the qualities of an effective mathematics teacher.
2. "Teacher is a pivot around which the whole educational system moves." Discuss the importance of the mathematics teacher in the light of the above statement.

Need and Avenues of Continuous Professional Development

Introduction

We cannot teach mathematics effectively without a thorough understanding of content and knowledge of pedagogy, which includes acquiring knowledge and skills for integrating technology into curriculum, instruction and assessment. In this unit, we will discuss the concept of professional development programmes for mathematics teachers. The term professional development refers to a comprehensive, sustained and intensive approach to improving teachers' effectiveness in raising learners' achievement. Such programmes help teachers align their teaching techniques with the needs of their learners and thereby ensuring better learner performance. We will discuss various ways of promoting professional development among mathematics teachers such as participation in seminars, conferences, online sharing, membership of professional organizations, etc. This unit will also shed light on the concept of teachers as a community of learners, where they are actively and intentionally involved in constructing knowledge together and disseminating it among their collective group. Mathematics teachers act as researchers all the time in their classrooms. Action research and innovation in teaching mathematics are part and parcel of the job description of teachers.

Need of Professional Development for Mathematics Teachers

Mathematics is often referred to as the science of numbers, quantities and shapes and, the relations between them. Like every other branch of science, mathematics is also constantly evolving. With technology growing by leaps and bounds, the means available to teachers for improving the teaching-learning process also grow. As teaching practitioners, mathematics teachers need to keep abreast of all such developments pertinent to their learners. Hence, the need for developing and promoting professional development among mathematics teachers arises.

The term professional development refers to an in-service training to upgrade the knowledge base and skills of the trainees. In the context of teachers, professional development facilitates an in-service training wherein the teachers get an opportunity to upgrade their content knowledge and pedagogical skills. Usually, it is in a formal setting, though at times it can be provided informally. It is essentially aimed at improving the overall efficacy of the teaching-learning process by enhancing the potentials of the teachers, and ultimately, the learners' performance.

Professional development is essentially a training provided to teachers during the course of their service period to promote their capacities in various aspects of content and pedagogy. To ensure learners' bright future, it is vital to support a cycle of continuous professional growth for teachers

Mathematics teachers are all the time hard pressed to make the subject an interesting and appealing one for their learners at the secondary and senior secondary level. The intricacies of mathematics begins to get more and more complex and the mathematics teachers find it increasingly difficult to retain the interest of the learners in the subject. It is a crucial stage for

learners as it prepares them for higher education and also for their future profession. In order to strengthen the educational delivery models practiced by the mathematics teachers, especially at the secondary and senior secondary level, it is vital that the teachers undergo professional development programmes from time to time so that they are better equipped to deal with the growing challenges of their classrooms.

Professional development programmes for mathematics teachers are designed to be comprehensive and rigorous, equipping the teachers with a wide-ranging set of pedagogical skills to become more effective in their classrooms. Organizing and participating in such professional development programmes help the teachers in the following ways:

- Become more attune to the latest developments in the field of mathematics
- Develop greater commitment towards their learners
- Create a platform for sharing professional experiences
- Work towards continuous improvement of their teaching skills
- Keep abreast of skills required to adapt to ever-changing learners' curriculum
- Tailor their teaching content according to the needs of their learners
- Acquire knowledge and skills for integrating technology into everyday teaching practices
- Develop the knowledge and skills which they need to address learners‘ learning challenges more effectively
- Integrate the latest research findings and teaching models to achieve better learning outcomes
- Apply action research findings in diverse classrooms

Professional Development Avenues for Mathematics Teachers

Professional development involves adopting a variety of strategies to ensure that teachers continue to strengthen their pedagogical skills throughout their career. In formal settings, professional development programmes are often disseminated through seminars, conferences, workshops, etc. Sometimes, professional development also occurs in informal conditions through interactions across online platforms, peer group discussions, etc. The common modes of professional development are:

- Individual reading and/or research
- Peer group discussions focused on a common topic or area of interest
- Observation - teachers observing other teachers so as to improve their own skills
- Expert advice an expert teacher advising colleagues
- Group meetings to learn a new strategy or teaching skill
- Adopting reflective and exploratory practices

- Staff meetings
- Online courses
- Short-term courses designed by experts at local/regional/national level
- Seminars/conferences/workshops to learn from a variety of expertise from around state or country
- Membership of professional bodies to keep oneself updated with the latest trends, practices and issues pertaining to the field of study

Seminars

Seminars are formal presentations by one or more experts in which the participants are encouraged to discuss a particular area of interest. In the context of professional development, it may be described as an occasion when a teacher or expert and a group of teachers meet to interact and discuss the intricacies of a chosen topic or subject. Mathematics as a subject is very interesting as well as challenging. Sharing experiences with other teachers and listening to experts on the concerned topics help the teachers evolve into better practitioners of teaching skills and improve learners' achievement. Seminars may be organized at the local, regional, state, national or international levels depending upon the needs and requirements of the teachers as well as the resources available. These seminars help the teachers update their knowledge base regarding latest developments in the field of mathematics and pedagogical skills being used. They also provide a platform for teachers to interact with experts to discuss and evolve new, effective strategies to deal with difficult classroom situations. By attending seminars, not only do the teachers themselves benefit from the discussions, but also their colleagues stand to gain from a summary of what one has learned and obtain copies of relevant documents.

These days with technological advancements, many a times the teachers find themselves attending webinars instead of seminars. Short for Web-based Seminar, a webinar is a seminar that is transmitted over the internet using video conferencing software.

A key feature of a webinar is its interactive elements 6 the ability to give, receive and discuss information in real-time. Using webinar, software participants can share audio, documents and applications with webinar participants. A webinar has all the advantages of a seminar without the limitations of actually being physically present at the venue. This is a great advantage for participants, especially in a profession like teaching, which leaves very little opportunity for the teachers to spare time for attending such seminars.

For example, a seminar on 'New Developments in Mathematics Teaching' inspires Mr. X to adopt constructive approach in his classroom. As per constructive belief, knowledge is actively created or invented by the child, not passively received from the environment. Learners create new mathematical knowledge by reflecting on their physical and mental actions. Ideas are constructed or made meaningful when learners integrate them into their existing structures of knowledge. Applying this principle in the classroom, the teacher Mr. X asks the learners to construct a triangle, where information about all three distances and all three angles has been given. Next, the teacher asks the learners to construct the next triangle, with fewer specifications, with less than six. Slowly, with each step, the learners learn to construct triangles with minimum information.

Conferences

Conferences may be described as formal meetings in which a number of people gather to talk about their ideas or problems related to a particular topic, usually for a few days. It is a large gathering of individuals or members of one or more organizations, for discussing matters of shared interest. Organizing conferences involving members of the teaching community is an effective way to render professional development to the teachers. Here, peer group discussions, shared experiences regarding classroom behaviours and results of action research, adopting reflective practices are some fruitful techniques that ultimately lead to the participants becoming better teachers.

For example, Mr. X attends a conference on 'Methods for Improving Mathematical Reasoning' among Secondary Level Learners. Here, he comes across many other secondary level mathematics teachers facing similar predicament - how to improve learners' reasoning abilities? In algebra, encouraging learners to think how changing an input (x) in an equation would affect the output (y). Discussing among themselves, the teachers realize that in order to solve such problems, the learners need to clearly see the abstract linear relationships. They all concur that explaining in simpler language, sometimes in the local language, would be beneficial. Upon coming back, Mr. X tries to adopt this technique in his classroom by explaining the algebraic word problems in Hindi and then asking the learners to solve the equations. He gets much quicker and better responses from the learners.

Online Sharing Communities

With the advancement of technology, sharing of resources and experiences among like. Minded people through online platforms has become a norm. Teachers as a community have a lot of common interests - issues related to learners, curriculum content and its implementation, new technologies available to the teachers, etc. Online platforms such as blogs, Facebook communities, etc. provide a good opportunity to the teachers to share their own experiences among a peer group, hold discussions on pertinent topics, participate in online seminars (known as webinars), even avail of online training courses aimed at enriching their knowledge and skill base. Nowadays, online sharing has gained popularity among the members of teaching community owing to its ease of use, versatility, global reach and cost effectiveness. Teachers are now transitioning to the online medium as a means for professional development. More and more teachers are looking to these online resources for solutions to their teaching issues and/ or training needs.

For example, The Math Forum is a community of teachers, mathematicians, researchers, learners, and parents using the internet to learn math and improve math education. The forum offers a wealth of problems and puzzles, online mentoring, research, team problem solving, collaborations and professional development. It helps educators share ideas and acquire new skills.

Membership of Professional Organizations

A professional organization (also referred to as a professional body), is usually a non-profit organization which seeks to further a particular profession, the interests of individuals engaged in that profession and the public interest. In the field of education, there are numerous professional organizations that are actively involved in facilitating better teaching-learning conditions and learners' achievement. With teachers juggling a multitude of roles,

both in professional as well as personal capacities, joining a professional organization ought to be a high priority for them. These professional organizations offer a plethora of benefits such as exclusive online resources, networking opportunities, free or discounted publications, chance to update their knowledge of business and trade basics or acquire new skills through seminars, workshops, conferences and online courses, develop mentoring relationships with more experienced teachers, etc.

The Association of Mathematics Teachers of India (AMTI) is one such organization in India that aims to assist teachers of mathematics at all levels in improving their expertise and professional skills for making mathematics interesting and enjoyable, and disseminates new trends in Mathematics Education, For example, the Association of Mathematics Teachers of India (AMTI) has been organizing seminars, conferences, workshops in various parts of the country to meet and deliberate on important issues in Mathematics Education, particularly at school level. Besides, the usual inauguration and valedictory functions, there are endowment/memorial lectures on applied mathematics, methodology and history of mathematics, group discussions, exhibitions, recreations in mathematics, paper presentations, learner's sessions, quiz- written and oral, Distinguished Mathematics Teacher Awards, etc.

Practice Questions

1. What is professional development? What is the need for professional development of mathematics teachers?
2. . What are the benefits of professional development for teachers?
3. How do seminars and conferences promote professional development?
4. How does online sharing help teachers in their professional development?

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Contribution of Mathematicians

INTRODUCTION

Mathematics is a subject which tends to lack at increased affective domain of learning with its content. But, affective domain is determinant of long lasting focus and consistency in leaning of a subject. Every mathematics teacher should not just focus on cognitive and psychomotor content of the subject. He/she should try to evolve emotional attachment of student with the subject. Knowing roots or history of a concept always helps in creating the awe factor among learners. Every mathematics student and teacher must know the contributions of great mathematician and appraise their efforts, which simplified many puzzling situations not just for general population but also other scientists.

Aryabhatta

Aryabhatta, known as Aryabhatta First, a great Hindu Mathematician lived at Kusumapura or Pataliputara in ancient Magadha or modern Patna in Bihar state. From the writing of at Bhaskra AD (629), it appears that Aryabhatta took up the profession of a teacher. Bhaskra mentions the names of Pander Ranga-swami, Latadeva and Nisanka amongst those who learn astronomy at the feet of Aryabhatta.

At the age of 23 years, Aryabhatta wrote at least two works on astronomy:

1. Aryabhatta
2. Aryabhatta-Sidhanta.

The Aryabhatta deals with both mathematics and astronomy. It contains 121 stanzas in all, and is marked for brevity and conciseness of composition. Like the yoga-darshan of Patanjali, the subject-matter of the Aryabhatta is divided into 4 chapters called Pada (or section).

- **Pada - 1 (viz., Gitika Pada)** consisting of 13 stanzas of haste definitions of important astronomical parameters and tables.
- **Pada - 2 (viz., Ganita Pada)** consisting of 33 stanzas deals with mathematics. The topics are geometrical figures with their properties and mensuration, series, linear and quadratic equations, methods for extracting the square roots, the cube roots, etc.
- **Pada - 3 (viz., Kalakriya Pada)** consisting of 25 stanzas deals with the true position of sun, moon and planets.

- **Pada - 4 (viz., Gola Pada)** consisting of 50 stanzas deals with the motion of sun, moon and planets on the celestial sphere.

Contribution of Aryabhatta

1. According to him, circumference: diameter = π = 3.14116, which is equivalent of saying that π = 3.1416.
2. Aryabhatta does not believe in the theory of creation and annihilation of world. For him, time is continuous process, without beginning and end (anadi and ananta). In the Smrits, as also in the Surya-Sidhanta, we have the following pattern of time divisions:

1 Kalpa = 14 Manus

1 Manu = 71 Yugas

1 Yuga = 42,20,000 years

3. Aryabhatta is probably the earliest astronomer to use the radian measure of 3438' for the radius of his circle.
4. 4. He made a notation system in which digits are denoted with the help of alphabet numerals, e.g.,

(ka) denotes 1, (kha) denotes 2, etc.

(Aa) denotes 100, (Aaa) denotes 101, (E) denotes 102, etc.

5. In the field of geometry, he gave formulae for calculating the area of different figures, e.g., area of square, rectangle, triangle, rhombus, circle, etc.
6. He also gave methods for the construction of various figures, e.g., triangle, rectangle , etc.
7. In Aryabhatta, he gave the methods of addition, subtraction, division and multiplication of and simple and compound algebraic quantities, e.g., he gave the following formulae: $(a+b)^2 = a^2 + b^2 + 2ab$.
8. He gave the following formulae to find the sum of AP series, if a and I are the first and last term of AP series, d is the common difference and n is the numbers of term extending from $(p + 1)^{th}$ term to $(p + n)^{th}$terms.

$S = n\left(a + \left(\frac{n-d}{P} + P\right)d\right)$ $\qquad$ $S = \frac{n(a+1)}{2}$

9. He tried to solve indeterminate linear equations as ax + by + c = 0 by the method of continued fractions.

10. In geometry, he gave the statement of the Pythagoras in the following form:

The square of the Bhuja (base) plus the square of the K (perpendicular) is the square of the Karne (Hypotenuse).

Ramanujan

Srinivasa Ramanujan Aiyangar, the remarkable mathematician genius, was a member of a Brahmin family in somewhat poor circumstances in Tanjore district of Madras Presidency.

Ramanujan's curiosity was to know the highest truth in mathematics. One day, when his teacher was explaining to the class that any quantity divided by it was equal to unity, he said to have stood up and asked, if zero divided by 0 also was equal to unity. It was at about this time that he mastered the properties of the three progressions. It was in the fourth form (4th year after primary) that he did each and every problem of trigonometry meant for degree classes without any aid.

It was in 1903, while he was in the sixth form, on a momentous day for Srinivasa Ramanujan, a friend of his secured for him the loan copy of Carr's Synopsis of Pune Mathematics from the library of the local Government College. It was this book that awakened his genius. He verified many of the results in the book and discovered many new results of his own.

He first devised some methods of constructing Magic numbers, then he brunched off to the Geometry and later to Algebra. In Algebra, he obtained many new series. In December 1903, he passes the matriculation examination of the Madras University and won the Subramanayam Scholarship which is generally awarded for proficiency in English and Mathematics.

He began correspondence with Prof. G.H. Hardy, a leading mathematician of his time. In his first letter, he attached 120 theorems of his own creation. Out of these, Prof. Hardy selected 15 theorems related to Hypergeometry, Elliptic Integral, Divergent Series, etc. about whom Prof. Hardy himself was unaware.

He reached Cambridge in April 1914 and was admitted into Trinity College. Prof. Hardy helped him to learn modern mathematics so as to acquaint himself up-to-date development in the field of mathematics.

Contribution of Ramanujan

1. He used to entertain his friends with his theorems and formulae, even in those early days. He had extraordinary memory and could easily repeat the value of pie, e, etc. to any number of decimal places. He remembered the idiosyncrasies of every one of first 10,000th integers.
2. 2. Taking all the numbers containing an odd number of dissimilar prime divisors, viz., 2, 3, 3 7, 11, 13, 17, 19, 23, 29, ..., he discovered that:

(a) the number of such numbers is less than n = $\frac{3}{\pi^2}$

(b) $\frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \frac{1}{11^2} + \frac{1}{31^2} = \frac{9}{2\pi^2}$

3. Ramanujan caught the attention of Prof. Hardy by sending 120 theorems on divergent series. About these theorems, Prof. Hardy wrote "A single look at them is enough to show that they could only be written by a mathematician of the highest class".
4. He was having great insight into algebraic formulae, transformation of infinite series and continued fraction.
5. He developed a formula for the partition of any number which can be made to yield the required result by a series of successive approximations. For example, take the case of 3. There have been alternative ways to write 3 + 0,1 + 2, 1 + 1 + 1, etc. We may easily verify that there are no other ways of 7 partitioning this number, if we do not wish to use fractional numbers.
6. On the theory of numbers. Goldbach's conjecture is one important work of Ramanujan. According to him, even integer greater than 2 is the sum of two primes, i.e., number having no divisors. Therefore, 4 is the sum of two primes 2; 2, 6 of the primes of 3; 3, 8 of the primes 3; 5, 10 of the primes 5 and 5 so on.

The Indian mathematician, Ramanujan although died in a young age, but caught the attention of the mathematical world by his remarkable work in the field of mathematics

Euclid

No definite information is available about the birthplace and life of Euclid. Most probably, he got his education in Plato's academy at Athens. But it is certain that he taught mathematics about 300 BC in the Royal School at Alexandra at Egypt.

Contributions of Euclid

1. He wrote geometry test which no one has written ever before, especially for those mature persons who needed no introductory work to convince than of the value of the subject of geometry. He collected all the confused pieces of vast mathematical puzzles and geometrical facts known in his days, arranged the various theorems in proper order, improved their proofs where necessary and added theorems he himself had thought out.
2. Euclid had written a book entitled "Elements" consisting of thirteen books. The first English translation of the work from the Greek appeared in the rear 1570. Since then, many more editions of his book appeared in the market. The popularity of Euclid's Elements is evidenced by the fact that it was one of the first works to be translated from Greek to various languages.
3. The concepts needed in each book were defined at the beginning Postulates and common notions {hypotheses} were stated. His elements were given or specification of what is to be proved, the construction needed if any, the proof, conclusion, and the condition under which the problem was impossible.
4. Below is given the content of the subject-matter of the thirteen books of the Elements:

1. Triangles, perpendiculars, parallel areas of rectilinear theorem, areas of rectilinear figures and the Pythagorean theorem
2. Transformation of areas and algebraic identities
3. Circles, chords and tangents
4. Construction of regular polygons
5. Proportion
6. Idea of proportion applied to similar figures
7. Even-odd numbers and numerical theory of proportion
8. Continued proportion
9. Number theory
10. Irrational
11. Solid geometry
12. Method of exhaustion used to show that circles are proportional to their diameter, etc.

13. Regular solids

5. He discovered a method of dividing an angle into three equal parts.
6. He devised a method of obtaining square from a circle
7. He found that prime numbers are infinite.
8. Euclid wrote another book entitled "Phenomena" which deals with the celestial sphere and contains twenty-five theorems on geometry.

Although Euclid is criticized by many mathematicians of modem time for rigorous and unscientific proofs of his theorems, but at the same time, he is also considered to be the foremost geometrician of his time who has created Geometry.

Pythagoras

Pythagoras was a classical Greek mathematician and philosopher. He was considered to be the founder of the movement called Pythagoreanism. A lot of his work was stored in the form of written discourse centuries after he lived. Hence, not much reliable information had been gathered on that front.

He was born in 570 century BC in the island of Samos. Pythagoras travelled to Greece and Egypt, and then moved to Croton where he established his school. He made contribution to the field of science and mathematics. He is recognized for presenting the noteworthy Pythagorean theorem. The account of his mathematical and philosophy career is tentative. Hence, the historians shared doubts about his true contributions to mathematics and philosophy. It is speculated that much of his notable works was actually accomplished by his successors and contemporaries. Pythagoras was the first one to refer himself as a philosopher and his ideas left a profound impact on Plato and helped shape the Western philosophy.

Owing to the fact that there was not much known about Pythagoras, myths were associated with him. It was Neoplatonist writers who came up with details about him. Isocrates and Herodotus believed that Mnesarchus was his father, who was a wealthy merchant and a gem engraver. He migrated to Greek colony of Croton around 530 BC, where he established a school of philosophy which was based on simple way of life as dictated by him. In addition to that, Pythagoras got involved in the Greek political affairs and acquired some influence. Due to political upheaval and unrest with the neighbouring colony of Sybaris, most of his disciples flee the colony and he too had to flee prior to the outbreak of civil war. He found shelter in Metapontum where he spent the rest of his life

Some historians believed that a lady of Croton, Theano, was his wife, who was one of his pupils and a philosopher. The popular opinion on the subject of their children is that they had a son and three daughters. Furthermore, it is speculated that Pythagoras had met another eminent philosopher, engineer and mathematician of that time, Thales of Miletus while visiting Greece. He was considered the mastermind behind several scientific discoveries and solution to mathematical problems.

In fact, Pythagoras is accredited for discovering the famous theorem which was named after him, Pythagorean Theorem. It was mostly used by Babylonians and Indians. It was either him or his students who came up with the first proof. The credit for the proof of theorem is still disputed because of the uncanny nature of Pythagoras' school where all the students attributed their work to their teacher.

There is no apparent evidence that Pythagoras was himself behind the deed. There are some skeptics who believe that he was not a mathematical genius and his mathematical endeavors was just a myth constructed by Plato's followers who had ulterior motives. This conspiracy theory is considered till this day. Cicero's and Plutarch's writings mentioned Pythagoras' work, posthumously. While these theories might hold some truth, there are also some who believed that Pythagoras worked out musical notes through mathematics. He once heard a melody and decided that its construction must be scientific and mathematically calculated application to the instrument. There is a possibility that Pythagoras discovered the properties of string length.

Practice Questions

- Briefly describe the life sketch and contributions of the mathematician Aryabhatta or Euclid.
- Briefly describe the life sketch and contributions of the mathematician Ramanujan or Pythagoras.
- Briefly describe the life sketch and contributions of any one mathematician Aryabhatta, Euclid or Pythagoras.

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11	8th April, 2017	Pal Rajendra Education Trust Pal Rajendra B.Ed College **"Paradigm Shift in Education for the 21st Century" Quality Education For All**	National Seminar	ISBN 978-81-933440-6-4
12	19th & 20th December 2017	**Gokhale Education Society's** ISO 9001 : 2008 **"Inclusive Education & Brain Based Learning"**	State Level	ISBN 978-81-933310-3
13	10th March 2018	Scholarly Research Journal For Interdisciplinary Studies **Attitude and Perception of women who left career Opportunities For Family Priorities**	International Level	ISSN 2278-8808 UGC Approved _ Sr. No - 49366 Peer Reviewed Online ISSN 2278-8808 Impact Factor 6.177
14	17th March, 2018	**Vidyawarta International Multilingual Research Journal** 'Dynamic of Effective Classroom teaching' 'Providing Joyful Learning Experience for the Learners' "Metacognition: Thinking about Thinking	National Level	ISSN: 2319 9318
15	13th Oct, 2018	Sanshodhjan Chetna	State Level	UGC Approval no. 63299

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26	21st & 22nd January 2020	Diversitas – A Compendium Emerging from the two day Seminar titled Readusting the Inclusion Lens – Transcending the Exclusion Mindset **Creating An Inclusive Abled (Accessible) Environment Through Public Private Partnership**	State Level	Peer Reviewed
27	2020	International Journal for Innovative Research in Multidisciplinary Field	International Level	Peer Reviewed ISSN: 2455-0620
28	Sept - Oct 2022	(ERJ) Educreator Research Journal **"Creativity as a 21st Century Skill"**	National Level	Peer Reviewed ISSN:P-2455-0515 E – 2394-8450
29	May – June 2023	Scholarly Research Journal for Interdisciplinary Studies	State Level	ISSN 2319-4766

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30	30th Sep 2023	PCERP Seminar Proceedings Book 20	National Level	ISBN 978-81-964882-4-6
31	14-15 Oct 2023	CTPD INDIA CTPDCON – ERA – 2023 SOUVENIR	First International E - Conference	Online Journal ctpdcon@gmail.com https://us06web.zoom.us/j/87894242715?pwd=ABGl2NEICCxmgP86y4gj4QsvAhb3tB.1
32	Jan – Feb 2024 2nd March 2024	Educator Research Journal Online and Print Journal Seva Sadan College of Education , Ulhasnagar	National Conference	Online Print Journal ISSN P – 2455-0515 E -2394-8450
33	March 2024	B.Aadhar Current Trends in Multidisciplinary Research	International Conference	Peer – Reviewed ISSN 2278-9308 Impact Factor 8.632
34				